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**METHODS STUDIES TOWARD SIMPLIFIED
ROTOR-BODY DYNAMICS**

Part I of First Yearly Report under Contract NAS2-7613

**Prepared for the Ames Directorate, AMRDL,
at Ames Research Center, Moffett Field, California**

by

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June, 1974

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Preface to First Yearly Report under Contract NAS2-7613

Work under Contract NAS2-7613 started on July 1, 1973. It is a continuation of research conducted since February 1, 1967, under Contract NAS2-4151. Phase VII-A, B and C Reports of June, 1973, titled, "Concepts for a Theoretical and Experimental Study of Lifting Rotor Random Loads and Vibrations", are the final reports under Contract NAS2-4151 and list the 9 preceding reports and 11 published articles and papers prepared under the contract.

Meanwhile 2 further papers generated under this Contract have been published:

Crews, S. T., Hohenemser, K. H. and Ormiston, R. A., "An Unsteady Wake Model for a Hingeless Rotor", Journal of Aircraft Vol. 10, No. 12, Dec. 1973, pp. 758-760.

Hohenemser, K. H. and Prelewicz, D. A., "Computer Experiments on Periodic Systems Identification Using Rotor Blade Transient Flapping-Torsion Responses at High Advance Ratio", AHS/NASA Ames Specialists Meeting on Rotorcraft Dynamics, Moffett Field, California, February 1974.

The first paper which has been generated under the new Contract NAS2-7613 is:

Hohenemser, K. H. and Yin, S. K., "On the Use of First Order Rotor Dynamics in Multiblade Coordinates", presented at the 30th Annual National Forum of the American Helicopter Society, May 1974, Preprint 831.

II

The research goals stated in Contract NAS2-7613 are

- (a) Assess analytically the effects of fuselage motions on stability and random response. The problem is to develop an adequate but not overly complex flight dynamics analytical model and to study the effects of structural and electronic feedback, particularly for hingeless rotors.
- (b) Study by computer and hardware experiments the feasibility of adequate perturbation models from non-linear trim conditions. The problem is to extract an adequate linear perturbation model for the purpose of stability and random motion studies. The extraction is to be performed on the basis of transient responses obtained either by computed time histories or by model tests.
- (c) Extend the experimental methods to assess rotor wake-blade interactions by using a 4-bladed rotor model with the capability of progressing and regressing blade pitch excitation (cyclic pitch stirring), by using a 4-bladed rotor model with hub tilt stirring, and by testing rotor models in sinusoidal up or side flow.

The first yearly report under Contract NAS2-7613 is subdivided into 3 parts, whereby Parts I, II, and III are related to the research goals (a), (b), and (c) respectively. The authors and titles of the three parts are:

III

Part I, Hohenemser, K. H. and Yin, S. K., "Methods Studies Toward Simplified Rotor-Body Dynamics".

Part II, Hohenemser, K. H. and Yin, S. K., "Computer Experiments in Preparation of System Identification from Transient Rotor Model Tests".

Part III, Hohenemser, K. H. and Crews, S. T., "Experiments with a Four-Bladed Cyclic Pitch Stirring Model Rotor".

Part I considers a number of simplifications in rotor-body dynamics and applies the various analytical models to a hypothetical compound hingeless rotorcraft with and without feedback into cyclic and collective controls.

Part II deals with the problem of rotor parameter identification from noise polluted transient blade flapping responses. Computer experiments are used in order to gain some insight into the efficiency of various identification schemes to be later applied to rotor model flapping transients.

Part III summarizes the test results obtained with the 4-bladed cyclic pitch stirring model rotor. The analytical blade flapping responses without considering the rotor wake are compared to the measured responses which include the wake-blade interactions.

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Abstract

This report is directed to the problem of developing an adequate but not overly complex linear flight dynamics analytical model of a rotorcraft to study stability, control, gust and random turbulence responses. Since the conventional flight dynamics analysis using quasisteady rotor derivatives is adequate for the long period modes like the phugoid mode, only short time responses are considered here, where rotor-body coupling is of importance. Thus the body motion consists of pitch, roll and vertical motion, omitting linear longitudinal and lateral and yaw perturbations. Five analytical models of varying degree of sophistication are applied to a hypothetical hingeless compound helicopter operating up to .8 rotor advance ratio. Stability and response data are obtained for the basic helicopter and for the vehicle with two simple control feedback systems. The least sophisticated analytical model which produces adequate results is determined for the various tasks. The random vehicle response to atmospheric turbulence shows a remarkable attenuation with either of the two control feedback systems.

METHODS STUDIES TOWARD SIMPLIFIED
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Part I of First Yearly Report under Contract NAS2-7613

<u>Table of Contents</u>	Page
Nomenclature	1
Introduction	3
Basic Rotorcraft Description	5
Feedback Systems	8
Rotor/Body Equations of Motion	10
System Simplifications	12
Effects of Rotor/Body Coupling on Flapping Stability	15
Eigenvalue Comparisons	16
Step Control Response Comparisons	20
Step and Random Gust Response Comparisons	23
Conclusions	26
References	31
Figure Captions	33
Figures	34-56

Nomenclature

B	tip-loss factor
E[...]	expected value of ...
F	state matrix
I	moment of inertia
K	feedback constant
L	right rolling moment over A/C rolling inertia, unit Ω^2 ; also turbulence scale length
M	nose-up pitching moment over A/C pitching inertia, unit Ω^2
N	blade number
P	blade elastic first flap-bending frequency when rotating, unit Ω , also covariance matrix
R	rotor radius
Z	down normal force over A/C mass, unit $R\Omega^2$
m	rotorcraft mass
m_b	blade mass
n	zero mean white noise
p	right rate of roll, unit Ω
q	nose-up rate of pitch, unit Ω
t	time, unit $1/\Omega$
w	down normal velocity, unit $R\Omega$
β	blade flapping angle defined by line from rotor center to blade tip
γ	blade Lock number referred to rotor center

δ	control input
θ	blade pitch angle
λ	up gust velocity, unit $R\Omega$
μ	advance ratio
ξ	real part of root
σ	standard deviation
τ	time constant (time unit $1/\Omega$)
ϕ	control phase angle
ψ	blade azimuth angle
ω	frequency, unit Ω
Ω	rotor angular speed, rad/sec

Superscripts:

.	time derivative, time unit $1/\Omega$
T	transposed matrix

Subscripts:

a	augmented, or acceleration
i	integral feedback for rotor tilting
x	about longitudinal axis
y	about lateral axis
I	forward cyclic
II	left cyclic
o	collective

Part I

Introduction

In Reference [1] stability and short time responses of a winged hingeless rotorcraft are studied with a linearized system of equations which includes 3 rotor modes - coning, regressing and progressing - , which includes periodic coefficients and which includes first blade mode elastic deflections using the methods of References [2] and [3]. Three control feedback systems were considered: Coning feedback, proportional cyclic feedback and a combined feedback. It was found that all three feedback systems - particularly the last one - were effective in alleviating control over sensitivity, control and damping cross coupling, pitch divergence and gust sensitivity.

The results are useful as trend studies but are only approximate because inplane blade dynamics, rotor wake dynamics and horizontal rotor forces have been omitted. The justification for omitting inplane blade dynamics is that inplane blade oscillations affect the body motions only in a narrow frequency band centered at the air resonance frequency. This fact evolved rather clearly in recently obtained unpublished frequency responses of the BO-105 helicopter, where the air resonance frequency is $.35\Omega$, which is at the upper end of the flight dynamically important frequency spectrum. Coupling

of a high gain feedback system or of the human controller with the air resonance mode may under certain conditions destabilize this mode, or at least cause high inplane blade loads. Apart from this special problem the effect of blade inplane dynamics on flying qualities of rotorcraft is expected to be usually small.

The justification of omitting rotor wake dynamics is that an equivalent reduced Lock number can approximate these effects, at least in the low end of the frequency spectrum which is of main importance in flight dynamics, see for example Reference [4]. Finally, the justification for omitting horizontal rotor forces is that their moments with respect to the aircraft C.G. are much smaller than the moments transmitted from the blades to the hub, at least for the rather high blade flapping frequencies assumed for the numerical examples. Such high flapping frequencies can be expected for unloaded slowed rotor operation in cruising of compound rotorcraft. For the short time responses the longitudinal and lateral linear perturbation motions of the rotorcraft have been neglected anyway, so that horizontal forces are not needed in the analysis.

The question studied in this report is whether simplifications in the rotorcraft representation of Reference [1] can be applied without introducing sizeable errors. The motivation to simplify the analytical model as much as feasible is not so much to save computer effort in establishing time

histories or stability characteristics, but rather to obtain a better visibility for the essential parameter effects. Another motivation is related to the fact that system identification algorithms and random response analyses do tend to require substantially more computer effort for increasing order and for time variability of the system. While the basic compound rotorcraft treated in this report is the same as in Reference [1], different feedback systems have been assumed in order to cover a wider range of such systems. This report is a more elaborate version of Reference [5], containing more analytical details and more numerical data.

Basic Rotorcraft Description

Though the same rotorcraft is used as an example in Reference [1], a brief description is given here for sake of completeness. The hingeless rotorcraft is the compound type, cruising with reduced rotor speed in an unloaded rotor condition. It is assumed that the rotorcraft performs a uniform forward motion and is restrained in yaw and side motion. The body is free to pitch, roll and move vertically. Thus we have added roll to the usual short time longitudinal flight dynamics because pitch and roll are coupled through the rotor modes. The classical phugoid, dutch roll and spiral modes cannot occur with the assumed restraint.

These modes are little modified by coupling with the rotor modes and reasonable approximations can be obtained with the conventional derivative analysis.

Due to the reduced rotor speed the blade flapping frequency in the rotating reference system is assumed to be rather high, $P = 1.2$. This assumption leads to a highly unstable basic rotorcraft and will impose severe demands on the selected control feedback systems. The stiff rotor also simplifies the analysis since in the first approximation inplane rotor forces need not be considered because of the relative smallness of their moments about the rotorcraft c.g. as compared to the hub moments. For advance ratio $\mu = .8$ flap-bending flexibility of the blades and reversed flow effects are taken into account following Reference [3]. A single elastic blade flap-bending mode is used. For advance ratio $\mu = .4$ flap-bending flexibility and reversed flow effects are neglected. The effect of the rotor wake dynamics on rotor frequency responses in the linear range was studied in Reference [4] from which one can conclude that for the high frequencies of the coning and advancing tilting modes dynamic wake effects are small, while for the low frequency regressing tilting mode, the assumption of a quasi steady wake is a reasonable approximation. The effects of the latter can be approximated by a certain reduction in blade Lock number, see References [4], [10] and [11]. Values for the blade Lock number γ in the

numerical examples of this paper are 5 and 8, covering the range of current hingeless rotors.

The wing area is 6% of rotor disk area. The wing lift slope is assumed to be 4.5, which would include rotor-wing interference effects. The horizontal tail area in excess of that required to neutralize the instability of the fuselage without rotor is 1.5% of rotor disk area. Tail moment arm is 1.2R, tail lift slope including wing and rotor downwash effects is 1.8. Downwash lag effects on the tail are neglected. With these values and using the rotor radius as length unit and Ω as angular velocity unit, one obtains for the body derivatives at .8 and .4 advance ratio

	$\mu = .8$	$\mu = .4$
Wing roll damping	$L_p = -.0200$	-.0100
Tail pitch damping	$M_q = -.0075$	-.0038
Tail pitch stability	$M_w = -.0063$	-.0032
Wing plus tail normal damping	$Z_w = -.0288$	-.0144
Tail normal force from pitch rate	$Z_q = -.0031$	-.0015

Body and blade masses and inertia moments are related by:

$$I_x/I_b = 5, \quad I_y/I_b = 75, \quad m_b/m = .20, \quad R^2 m_b/I_x = .60$$

Data not shown here have also been obtained for $I_y/I_b = 25$ with essentially the same conclusions. The rotor is assumed to be 3-bladed. The control phase angle for the basic rotorcraft is 45° .

Feedback Systems

Two feedback systems are considered. The first, has rotor tilting plus body angular rate feedback with a large actuator lag. It was described in Reference [6] and its performance is almost identical to that of the Lockheed gyro control AMCS described in Reference [7], though the mechanization is different. Wind tunnel model tests with a variant of this system using an isolated hingeless rotor are described in Reference [8]. In the flight dynamics frequency range this feedback system emasculates the rotor with respect to tilting, so that angle of attack instability, control over sensitivity, control cross coupling, cross damping, and stick reversal in g maneuvers are avoided. The feedback equations in their linear form are

$$\dot{\theta}_I + \theta_I/\tau = -K_I \beta_I + \delta_I \cos \phi - \delta_{II} \sin \phi + K_I q \quad (1)$$

$$\dot{\theta}_{II} + \theta_{II}/\tau = -K_I \beta_{II} + \delta_I \sin \phi + \delta_{II} \cos \phi + K_{II} p \quad (2)$$

The values for the parameters used in the numerical examples are the following. The actuator time constant is $\tau = 10$, which is the time for 1.6 rotor revolutions. The rotor tilting gain is $K_I = .3$, the control phase angle is $\phi = 7.5^\circ$, the rate gyro gains are $K_I = K_{II} = 1$.

The second system has normal body acceleration feedback into collective pitch. Without fuselage aerodynamics this is equivalent to coning feedback. According to Reference [9] this system proved to be effective in substantially extending the speed range for acceptable handling characteristics of a hingeless rotorcraft. The feedback equation is

$$\ddot{\theta}_0 + \tau \dot{\theta}_0 = K_a(\dot{w} - q\dot{u}) \quad (3)$$

For the numerical examples the actuator time constant is $\tau = .5$, which is the time for .08 rotor revolutions. The gain is $K_a = 10$. In addition one needs the cyclic control relations

$$\theta_I = \delta_I \cos \phi - \delta_{II} \sin \phi \quad (4)$$

$$\theta_{II} = \delta_I \sin \phi + \delta_{II} \cos \phi \quad (5)$$

where the control phase angle is $\phi = 25^\circ$. The control phase angles for the basic rotorcraft and for the craft with the two feedback systems were selected in such a way as to minimize control cross coupling at $\gamma = 5$ over the entire flight range between 0 and .8 advance ratio. For the craft with tilting feedback the tail size was reduced to 1% of rotor disk area, since this feedback system is more effective than the normal acceleration feedback. Schematics of the two feedback systems are shown in Fig. 1

Rotor/Body Equations of Motion

One form of the linearized rotor/body equations of motion are given in Reference [1], Eqs. (23) to (35). Here a more developed form is given which is directly usable for programming. For advance ratio $\mu = .4$ rigid straight blades are assumed, which are elastically hinged at the rotor center. The equations for this case can be given in exact form, if the small reverse flow effects are neglected. For advance ratio $\mu = .8$ reversed flow and blade elasticity are not negligible. In this case a Fourier analysis of the periodic terms was performed retaining the first 7 terms of the Fourier series. The equations were transformed to non rotating body fixed coordinates in state variable form. The basic rotorcraft is a ninth order dynamic system, with the state variables $p, q, w, \beta_I, \dot{\beta}_I, \beta_{II}, \dot{\beta}_{II}, \beta_O, \dot{\beta}_O$. With tilting feedback according to Eqs. (1) and (2) one obtains an eleventh order dynamic system including the state variables θ_I, θ_{II} . With normal acceleration feedback according to Eq. (3) one obtains a tenth order dynamic system including the state variable θ_O .

At low advance ratios ($\mu < .5$), where rigid straight blades are assumed and the small reverse flow effects can be neglected, the linearized rotor/body equations of motion, for a three-bladed rotor, in terms of multiblade coordinates are

$$\begin{aligned}
& \ddot{\beta}_0 + (B^4\gamma/8)\dot{\beta}_0 + (B^3\gamma\mu/12)\dot{\beta}_{II} + (B^2\gamma\mu^2/16)\beta_I \sin 3t \\
& - (B^2\gamma\mu^2/16)\beta_{II} \cos 3t + p^2 \beta_0 - 1.5 (\dot{w} - \mu q) - (B^3\gamma\mu/12)p \\
& - (B^3\gamma/6)w = (B^4\gamma/8 + B^2\gamma\mu^2/8)\theta_0 - [B^3\gamma\mu/6 - \\
& (B^2\gamma\mu^2/16) \sin 3t]\theta_I - (B^2\gamma\mu^2/16)\theta_{II} \cos 3t
\end{aligned} \tag{6}$$

$$\begin{aligned}
& \ddot{\beta}_I + (B^4\gamma/8)\dot{\beta}_I + (p^2 - 1)\beta_I + (B^3\gamma\mu/6)\beta_0 + 2\dot{\beta}_{II} \\
& + (B^4\gamma/8 + B^2\gamma\mu^2/16)\beta_{II} + [(B^3\gamma\mu/6)\beta_I - (B^3\gamma\mu/12)\dot{\beta}_{II}] \\
& \cos 3t + [(B^2\gamma\mu^2/8)\beta_0 + (B^3\gamma\mu/12)\dot{\beta}_I + (B^3\gamma\mu/6)\beta_{II}]\sin 3t \\
& - \dot{q} - 2p - [B^4\gamma/8 + (B^3\gamma\mu/12)\sin 3t]q \\
& + (B^3\gamma\mu/12)p \cos 3t = -(B^2\gamma\mu^2/8)\theta_0 \cos 3t \\
& + (B^3\gamma\mu/6)\theta_I \cos 3t + [B^4\gamma/8 + B^2\gamma\mu^2/16 + (B^3\gamma\mu/6)\sin 3t]\theta_{II}
\end{aligned} \tag{7}$$

$$\begin{aligned}
& \ddot{\beta}_{II} + (B^4\gamma/8)\dot{\beta}_{II} + (p^2 - 1)\beta_{II} + (B^3\gamma\mu/6)\dot{\beta}_0 - 2\dot{\beta}_I + (B^2\gamma\mu^2/16 \\
& - B^4\gamma/8)\beta_I + [(B^3\gamma\mu/6)\beta_I - (B^3\gamma\mu/12)\dot{\beta}_{II}]\sin 3t - [(B^2\gamma\mu^2/8)\beta_0 \\
& + (B^3\gamma\mu/12)\dot{\beta}_I + (B^3\gamma\mu/6)\beta_{II}]\cos 3t - \dot{p} + 2q + (B^3\gamma\mu/12)q \cos 3t \\
& - [(B^4\gamma/8) - (B^3\gamma\mu/12)\sin 3t]p - (B^2\gamma\mu/4)w = [B^3\gamma\mu/3 \\
& - (B^2\gamma\mu^2/8)\sin 3t]\theta_0 - [B^4\gamma/8 + 3B^2\gamma\mu^2/16 - (B^3\gamma\mu/6)\sin 3t]\theta_I \\
& - (B^3\gamma\mu/6)\theta_{II} \cos 3t
\end{aligned} \tag{8}$$

$$\dot{p} - L_p p = 1.5 (I_b/I_x)(1 - P^2)\beta_{II} \quad (9)$$

$$\dot{q} - M_q q - M_w w = 1.5 (I_b/I_y)(1 - P^2)\beta_I \quad (10)$$

$$\begin{aligned} \dot{w} - \mu q - Z_w w - Z_q q - (m_b/m)[1.5 \ddot{\beta}_0 + (B^3 \gamma/6)\dot{\beta}_0 \\ + (B^2 \gamma \mu/8)(\dot{\beta}_{II} - \beta_I - p) + (B^2 \gamma \mu/8)\beta_I + (B \gamma \mu^2/8) \\ (\beta_I \sin 3t - \beta_{II} \cos 3t) - (B^2 \gamma/4)w] = (m_b/m)[-(B^3 \gamma/6 \\ + B \gamma \mu^2/4)\theta_0 + (B^2 \gamma \mu/4)\theta_I + (B \gamma \mu^2/8)(\theta_{II} \cos 3t - \theta_I \sin 3t)] \quad (11) \end{aligned}$$

At higher advance ratios, e.g. $\mu = .8$, where blade flexibility and reverse flow effects have been considered, the coefficients in the linearized rotor/body equations of motion depend on blade flapping mode shapes and flow regions, and can no longer be expressed explicitly in general forms in terms of advance ratio μ and blade tip loss factor B . Therefore the equations used in the numerical examples for $\mu = .8$ are not given here.

System Simplifications

The linear rotor perturbation equations (6) to (11) are of second order in β_I , β_{II} , β_0 and include periodic terms. We are considering 5 types of system modeling indicated in Table 1. Type 1 is the complete system including periodic terms, body

motions, multiblade accelerations and multiblade rates. Type 2 is the rotor periodic system obtained by rigidly restraining the body. Type 3 is the rotor/body constant coefficient system of second order in each multiblade coordinate obtained by omitting the periodic terms. With full body freedom this has been also called the 9 x 9 system since there are 6 body and 3 multiblade degrees of freedom. Type 4 is new and is introduced in this paper and in Reference [5]. The system is of first order in each multiblade coordinate. The periodic terms and the multiblade accelerations are omitted. Type 5 is of zero order in the multiblade coordinates; both multiblade accelerations and rates are omitted. This system is equivalent to the conventional 6 x 6 quasisteady derivative system.

Table 1

Five Types of System Modeling

Type No.	Periodic Terms	Body Motions	Accelerations $\ddot{\beta}_I \ \ddot{\beta}_{II} \ \ddot{\beta}_O$	Rates $\dot{\beta}_I \ \dot{\beta}_{II} \ \dot{\beta}_O$
1	X	X	X	X
2	X		X	X
3		X	X	X
4		X		X
5		X		

The effects of omitting the multiblade accelerations will be shown for the case of an isolated rotor in vacuum. The equations for the multiblade tilting coordinates β_I , β_{II} are

$$\ddot{\beta}_I + (P^2 - 1)\beta_I + 2\dot{\beta}_{II} = 0 \quad (12)$$

$$\ddot{\beta}_{II} + (P^2 - 1)\beta_{II} - 2\dot{\beta}_I = 0 \quad (13)$$

which are satisfied for

$$\beta_I = e^{i\omega t}, \quad \beta_{II} = i e^{i\omega t}, \quad \omega = 1 + P. \quad (14)$$

The solutions describe an advancing tilting mode with frequency $1 + P$ and a regressing tilting mode with frequency $P - 1$.

Omitting the accelerations $\ddot{\beta}_I$, $\ddot{\beta}_{II}$, the equations have only one root, $\omega = (P^2 - 1)/2$ which is approximately equal to $P - 1$ for not too large $P - 1$. Thus first order rotor dynamics in multiblade coordinates removes the advancing tilting mode with the high frequency $1 + P$ and yields a regressing tilting mode with approximately the same frequency as for the complete system of equations.

For the coning mode in hovering we have

$$\ddot{\beta}_0 + (\gamma/8)\beta_0 + P^2\beta_0 = 0 \quad (15)$$

Leaving out the acceleration $\ddot{\beta}_0$ reduces the coning degree of freedom to a first order system with spring and damping only. Note that what is called here first order rotor dynamics is not equivalent to omitting the $\ddot{\beta}$ term in the individual blade flapping equation. The terms omitted are the multiblade accelerations $\ddot{\beta}_I$, $\ddot{\beta}_{II}$ and $\ddot{\beta}_0$. Physically

this means that in an inertial frame of reference those blade inertia moments are neglected, which would occur at zero rotor speed, while the inertia moments due to rotation - the much larger gyroscopic moments - are retained.

Effects of Rotor/Body Coupling on Flapping Stability

We will consider here both the rotor/body periodic system and the isolated rotor periodic system. In Reference [1] it was shown that rotor/body coupling had little effect on the root curves for coning and proportional tilting feedback. Here the comparison is extended to the lagged tilting moment feedback expressed in Equations (1) and (2). The analysis makes use of the Floquet state transition matrix in the multiblade coordinate form and extracts the characteristic values from this matrix. The ambivalence in assigning frequency values makes it possible to use only the positive frequency half plane. For the 3-bladed rotor assumed in this study the flapping instability occurs as a coupled advancing and coning mode with frequency 1.5. Figure 2 shows the root plots for the rotor tilting feedback system with gain K_1 as parameter. The advance ratio is .8 and the cases of blade Lock number of 5 and 8 are shown. There is very little difference in the root curves for the rotor alone - dashed lines - and the rotor/body system, solid lines. However, the blade Lock number has a substantial effect, whereby the limiting gain is reduced from 5 to 8. From Figure 2 it appears that a stability analysis for the coupled advancing and coning mode can be

made with the rotor alone and rotor/body coupling will not affect the result very much.

For normal acceleration feedback the case of the rotor alone is not really comparable to that for the complete rotor/body system, since the fixed wing contributes substantially to the vertical damping thus improving the stability. Therefore, Fig. 3 shows only the case with rotor/body coupling for an advance ratio of .8. The instability of the coupled advancing and coning mode occurs in the same way as before with a frequency of 1.5. Increasing the blade Lock number from 5 to 8 reduces the limiting gain from about $K_a = 40$ to 30. The stability of the coupled advancing and coning mode increases with decreasing advance ratio, where the coefficients of the periodic terms are smaller. The highest advance ratio is, therefore, the most critical one.

Eigenvalue Comparisons

Table 2 shows in the first column the eigenvalues for advance ratio $\mu = .8$ and blade lock number $\gamma = 5$. It shows in the second column the effects on eigenvalues of changing at $\gamma = 5$ the advance ratio from .8 to .4, and in the third column the effects of changing at $\mu = .8$ the blade lock number from $\gamma = 5$ to 8. The complete equations of the periodic system were used. The fourth column gives for $\mu = .8$ and $\gamma = 5$ the effects of omitting the periodic terms

in the multiblade equations, the fifth column shows the effects of omitting in addition the accelerations, and the 6th column shows the effects of omitting in addition the rates of the multiblade coordinates. The 3 sets of rows in Table 2 refer to the 3 cases of the basic rotorcraft, of the craft with rotor tilting, and of the craft with normal acceleration feedback. The coupled modes are named according to the predominant uncoupled modal content of each. As mentioned before, the basic rotor/body system is of 9th order, with tilting feedback of 11th order, with normal acceleration feedback of 10th order. With first order rotor dynamics in each multiblade coordinate the order of all systems is reduced by 3. With zero order rotor dynamics, representing the derivative approach, the order is reduced by 6.

To look into the effect of advance ratio, compare the first two columns of Table 2. The basic rotorcraft has at $\mu = .8$ a pitch divergence which is about 6 times more severe than at $\mu = .4$. The other eigenvalues change relatively little with μ . With tilting feedback the pitch divergence is replaced by a stable oscillation. One roll mode is more damped, the other less damped than without feedback. Coning and advancing modes show little change with feedback or with advance ratio. With normal acceleration feedback the pitch divergence is also removed, however here the pitch mode changes from two convergences to an oscillatory mode when going from $\mu = .4$ to $\mu = .8$.

To look into the effect of advance ratio, compare the first two columns of Table 2. The basic rotorcraft has at $\mu = .8$ a pitch divergence which is about 6 times more severe than at $\mu = .4$. The other eigenvalues change relatively little with μ . With tilting feedback the pitch divergence is replaced by a stable oscillation. One roll mode is more damped, the other less damped than without feedback. Coning and advancing modes show little change with feedback or with advance ratio. With normal acceleration feedback the pitch divergence is also removed, however here the pitch mode changes from two convergences to an oscillatory mode when going from $\mu = .4$ to $\mu = .8$.

- To look into the effect of blade Lock number at $\mu = .8$ compare the first and third columns of Table 2. The basic rotorcraft shows a large increase in pitch divergence from increased γ . With tilting feedback the blade Lock number has little effect on the pitch mode, while some of the other modes are strongly affected by γ . The same is true for the craft with normal acceleration feedback.

To look into the degree of approximation provided by the 3 constant system models, compare the first column with the last 3 columns of Table 2. The pitch mode eigenvalue is in most cases reasonably well approximated even by the zero order system. The two roll mode eigenvalues are well approximated by the first order system in each multiblade

coordinate, but considerably in error for the zero order system. The second order system provides reasonably accurate eigenvalues for all modes, however data not shown here indicate that with increasing blade Lock number increasing effects of periodicity occur.

Step Control Response Comparisons

All transient responses shown in Figs. 4 to 12 are given for a non-dimensional time period of 40 which is the time for 6.3 rotor revolutions. One can assume that within this short time period the effects of the omitted phugoid, dutch roll and spiral modes will be relatively small as compared to the effects of the short period pitch and roll modes. The figures show 4 response variables: The roll rate p , the pitch rate q , the normal velocity w , which in a body fixed reference system is proportional to angle of attack, and the normal acceleration $\dot{w} - qv$. In Figs. 4 to 10 the responses to both unit longitudinal and lateral control input are shown.

Fig. 4 is for the basic rotorcraft at $\mu = .8$, $\gamma = 5$, $\phi = 45^\circ$. The high degree of instability and the high control cross coupling is evident in spite of an effort to minimize the control cross coupling by selecting a proper control phase angle ϕ . The oscillations in the roll rate and normal acceleration responses have a frequency of 3 and are caused by the periodic terms. In a linear analysis these

3 per rev. oscillations for the 3-bladed rotor must be superimposed to the oscillations from the trim condition. Fig. 5, also for the basic rotorcraft at the same conditions, shows the responses to the unit control inputs for the constant first order and zero order systems. The first order system responses are very close to those of the periodic system in Fig. 4, except for the missing oscillations. Use of the zero order system leads to substantial errors.

Fig. 6 shows for the rotorcraft with tilting feedback at $\mu = .8$, $\gamma = 5$, $\phi = 7.5^\circ$ the responses to unit control inputs computed with the zero and first order systems. Pitch and roll rates stabilize within a short time period, the normal acceleration soon becomes concave downward. Control cross couplings are negligible. The zero order responses deviate initially from the first order responses, particularly in roll rate. For the remaining numerical examples the first order approximation was used. Fig. 7 shows for the rotorcraft with tilting feedback the effects of blade Lock number on the responses to unit control inputs for $\mu = .8$, $\phi = 7.5^\circ$. It is seen that this feedback system is not sensitive to changes in blade Lock number which agrees with the concept of "rotor emasculation".

Fig. 8 shows for the rotorcraft with normal acceleration feedback the effect of blade Lock number on the responses to unit control inputs for $\mu = .8$, $\phi = 25^\circ$. The improvement over the basic rotorcraft is very good for $\gamma = 5$. Pitch and roll rate stabilize rapidly, and the normal acceleration becomes concave downward after about 4 rotor revolutions. Control cross coupling is small. For $\gamma = 8$, the roll rate from longitudinal control input is large, indicating that $\phi = 25^\circ$ is not optimal.

The effects of advance ratio on the step control responses are shown in Figs. 9 and 10. Advance ratios $\mu = .4$ and $.8$ are assumed. Fig. 9 is for the rotor tilting moment feedback. It is seen that control cross coupling remains small for both advance ratios. The normal acceleration per unit control input is for $\mu = .4$ smaller than for $\mu = .8$. Fig. 10 is for the normal acceleration feedback. Here the case for $\mu = .4$ shows more control cross coupling than the case for $\mu = .8$, indicating that for this type of feedback the selection of a control phase angle which keeps the control cross coupling effects small over the entire flight regime, is somewhat of a problem which does not exist for the "emasculated" rotor with the rotor tilting feedback.

Step and Random Gust Response Comparisons

Gust responses are obtained from the equations of motions by replacing the normal velocity w by the sum of normal

body velocity and normal gust velocity $w + \lambda$, whereby λ is prescribed. For unit gust step inputs λ is equal to the unit step function. For random gust input the standard deviation σ_λ is assumed to be a unit step function of time. The standard deviations for roll, pitch, normal body velocity and normal body acceleration, σ_p , σ_q , σ_w , $\sigma_{\dot{w}-q\mu}$ respectively are then computed for the case of the rotorcraft encountering at time $t = 0$ a turbulence region with unit standard deviation σ_λ . Since the basic rotorcraft is highly unstable, a steady turbulence response state does not exist, however, the initial random responses can be compared to those with control feedback. The gust responses shown in Figs. 11 and 12 have again been obtained with first order rotor dynamics

The random response analysis follows Reference [12]. The "point" assumption is used, according to which the entire rotor disk experiences at a given time the turbulence velocity at the rotor center. The normal gust velocity is obtained by passing white noise through a first order filter

$$\dot{\lambda} + a\lambda = \sigma_\lambda (2a)^{1/2} n \quad (16)$$

where

$$a = 2\mu/(L/R) \quad (17)$$

The ratio of turbulence scale length L over rotor radius R is assumed to be 12, for example $L = 360$ ft, $R = 30$ ft. The rotor/body equations of motion written in multiblade state variable form are augmented by Eq. (16) with the additional state variable λ . The augmented state vector x_a has the components $\beta_o, \beta_I, \beta_{II}, p, q, w, \lambda$. The random gust response equations then read

$$\dot{x}_a = F_a x_a + G v \quad (18)$$

where F_a is the augmented state matrix and where G and v are given by

$$G = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ 0 & \cdot & \cdot & \cdot & (2a)^{1/2} \end{bmatrix} \quad v = \begin{bmatrix} n \\ \cdot \\ \cdot \\ \cdot \\ n \end{bmatrix} \quad (19)$$

n is zero mean white noise with standard deviation $\sigma_n = 1$. The equation for the augmented covariance matrix P_a is, see for example Reference [12],

$$\dot{P}_a = F_a P_a + P_a F_a^T + G G^T \quad (20)$$

For purposes of determining the response standard deviations we need only the unaugmented 6x6 covariance matrix without λ

$$P = R_{xx} = E[xx^T] \quad (21)$$

and the unaugmented state vector equation

$$\dot{x} = Fx \quad (22)$$

The standard deviations for the state variables σ_p , σ_q , σ_w are directly obtained from the diagonal terms of $P = R_{xx}$. The covariance matrix for the normal acceleration is

$$\begin{aligned} R_{aa} &= E[(\dot{w}-qu)(\dot{w}-qu)^T] \\ &= R_{\dot{w}\dot{w}} - 2uR_{\dot{w}q} + u^2R_{qq} \end{aligned} \quad (23)$$

R_{qq} is a component of the covariance matrix R_{xx} . $R_{\dot{w}q}$ is a component of

$$R_{\dot{x}x} = F R_{xx} \quad (24)$$

and $R_{\dot{w}\dot{w}}$ is a component of

$$R_{\dot{x}\dot{x}} = F R_{xx} F^T \quad (25)$$

Thus the standard deviation of the normal acceleration $\sigma_{\dot{w}-qu}$ can be computed from Eqs. (23) to (25).

Fig. 11 shows the unit step gust responses p , q , w , $\dot{w} - qu$ for $\mu = .8$ and $\gamma = 5$. Responses for the basic rotorcraft and for the craft with the control feedback systems are given. In addition, the q , w , and $\dot{w} - qu$ responses are also

shown without the rotor. The combination of body, wing and horizontal tail represents a stable configuration. The destabilizing effect of the rotor is clearly recognizable, also the stabilizing effects of the two feedback systems. The initial maximum normal acceleration is not much affected by the feedback systems, since a large portion of this normal acceleration has its origin in the fixed wing.

Fig.12 shows the standard deviations of the same 4 variables following entry into a turbulence region at $t = 0$. The feedback systems are very effective in reducing the random responses in roll and pitch rate. Without rotor the pitch rate standard deviation σ_q is almost as high as with rotor. Either of the two feedback systems brings a large reduction in random pitch rate response even as compared to the stable configuration without rotor. The normal acceleration standard deviation $\sigma_{\dot{w}-q_u}$ is also caused in part by the fixed wing. The feedback systems reduce the rotor contribution considerably, but the overall value is not decreased by a large amount. In alleviating normal accelerations from either step or random gusts, the normal acceleration feedback system is slightly more effective than the rotor tilting feedback system. The normal velocity standard deviation is highest without rotor and lowest for the craft without feedback. This is a consequence of the

rotor normal damping. With feedback systems rotor normal damping is reduced and intermediate values of normal velocity standard deviation obtained.

Conclusions

The following conclusions are based on results of a linear coupled rotor/body flight dynamics analysis for a hingeless compound helicopter operating up to .8 advance ratio. The hingeless rotor is rather stiff in flapping ($P = 1.2$), the fixed wing has 6% rotor disk area, the horizontal tail has 1.5% rotor disk area in excess of that required to neutralize the body instability. The conclusions are subdivided into those on analytical modeling and those on flying characteristics of the hypothetical rotorcraft.

Conclusions on Analytical Modeling

- The analysis of flapping instabilities due to interblade coupling from the control feedback systems requires the retention of the terms with periodic coefficients in the equations of motion. For rotor tilting feedback into cyclic pitch, rotor/body coupling effects are small and the isolated rotor analysis is adequate. For normal acceleration feedback into collective pitch, damping of the vertical motion by the fixed wing is beneficial and a coupled rotor/body flapping stability analysis is required.

- The analysis of transient responses to cyclic pitch step inputs including the terms with periodic coefficients, resulted in substantial N per rev. oscillations of the rigid body in roll rate and in normal acceleration. If these rigid body oscillations are of no interest, constant coefficient modeling can be used. Of the three constant coefficient models studied, the one using second order rotor dynamics in each multiblade coordinate - equivalent to what is sometimes called the 9 x 9 model - was found to be over-sophisticated, the one using first order rotor dynamics was found to be adequate, the one using zero order rotor dynamics - equivalent to the 6 x 6 or derivative model - can result in substantial errors. If the rotorcraft is stabilized by a feedback system, these errors are limited to the initial response and fade out with time. For an unstable craft the errors increase with time.

Conclusions on Flying Characteristics

- Following a cyclic pitch control step input the basic rotorcraft shows rapid divergence in roll rate, pitch rate, normal velocity, and normal acceleration. Large control cross coupling effects are unavoidable inspite of optimization of the control phase angle. Of the eigenvalues only that for a predominant pitch mode indicates divergence. Due to cross coupling a lateral control step input also leads to

rapid divergence. A vertical step gust leads to pitch divergence associated with considerable roll. Entering a region of vertical turbulence the standard deviations for roll rate, pitch rate, normal velocity and normal acceleration rapidly reach a temporary level from which they then gradually increase. About 60% of the normal acceleration turbulence response is caused by the fixed wing.

- The rotor tilting feedback into cyclic pitch, even with a reduced tail area of 1%, completely stabilizes the rotorcraft and removes the cross control coupling effects for both $\gamma = 5$ and 8 and for $\mu = .4$ and $.8$. In a step gust this feedback does not alleviate the first normal acceleration peak, however the normal acceleration soon approaches zero rather than increasing again due to pitch-up as for the basic craft. When penetrating a turbulence area the final level of standard deviations in roll and pitch rate are only 30 to 40% of the initial level for the basic craft. The standard deviation level for the normal acceleration is not reduced materially since the fixed wing provides the major contribution.
- The normal acceleration into collective pitch using now 1.5% tail area also completely stabilizes the rotorcraft. For blade Lock number $\gamma = 5$ and advance ratio $\mu = .8$ cross control coupling effects are negligible, for $\gamma = 8$ and

$\mu = .8$, or for $\gamma = 5$ and $\mu = .4$ using the same control phase angle there is substantial roll from longitudinal control input. Otherwise this feedback system provides flying characteristics very similar to those for the rotor tilting feedback system and leads to somewhat lower values of turbulence response in roll and in normal acceleration. It is rather remarkable that for both feedback systems the pitch rate turbulence response is lower than without rotor. The lifting rotor with its feedback system absorbs part of the pitch rate turbulence response the craft would experience without rotor.

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Figure Captions

- Fig. 1 Schematics of the Feedback Systems
- Fig. 2 Root Plots for Rotor Tilting Feedback, $\mu = .8$, $\gamma = 5$ and 8, Periodic System
- Fig. 3 Root Plots for Normal Acceleration Feedback, $\mu = .8$, $\gamma = 5$ and 8, Periodic System
- Fig. 4 Control Responses of Basic Rotorcraft, $\mu = .8$, $\gamma = 5$, $\phi = 45^\circ$, Periodic System
- Fig. 5 Control Responses of Basic Rotorcraft, $\mu = .8$, $\gamma = 5$, $\phi = 45^\circ$, First and Zero Order Systems
- Fig. 6 Control Responses with Rotor Tilting Feedback, $\mu = .8$, $\gamma = 5$, $\phi = 7.5^\circ$, First and Zero Order Systems
- Fig. 7 Effect of Blade Lock Number on Control Responses with Rotor Tilting Feedback, $\phi = 7.5^\circ$, $\mu = .8$
- Fig. 8 Effect of Blade Lock Number on Control Responses with Normal Acceleration Feedback, $\phi = 25^\circ$, $\mu = .8$
- Fig. 9 Effect of Advance Ratio on Control Responses with Rotor Tilting Feedback, $\gamma = 5$, $\phi = 7.5^\circ$
- Fig. 10 Effect of Advance Ratio on Control Responses with Normal Acceleration Feedback, $\gamma = 5$, $\phi = 25^\circ$
- Fig. 11 Step Gust Responses, $\mu = .8$, $\gamma = 5$, First Order System
- Fig. 12 Turbulence Responses, $\mu = .8$, $\gamma = 5$, First Order System

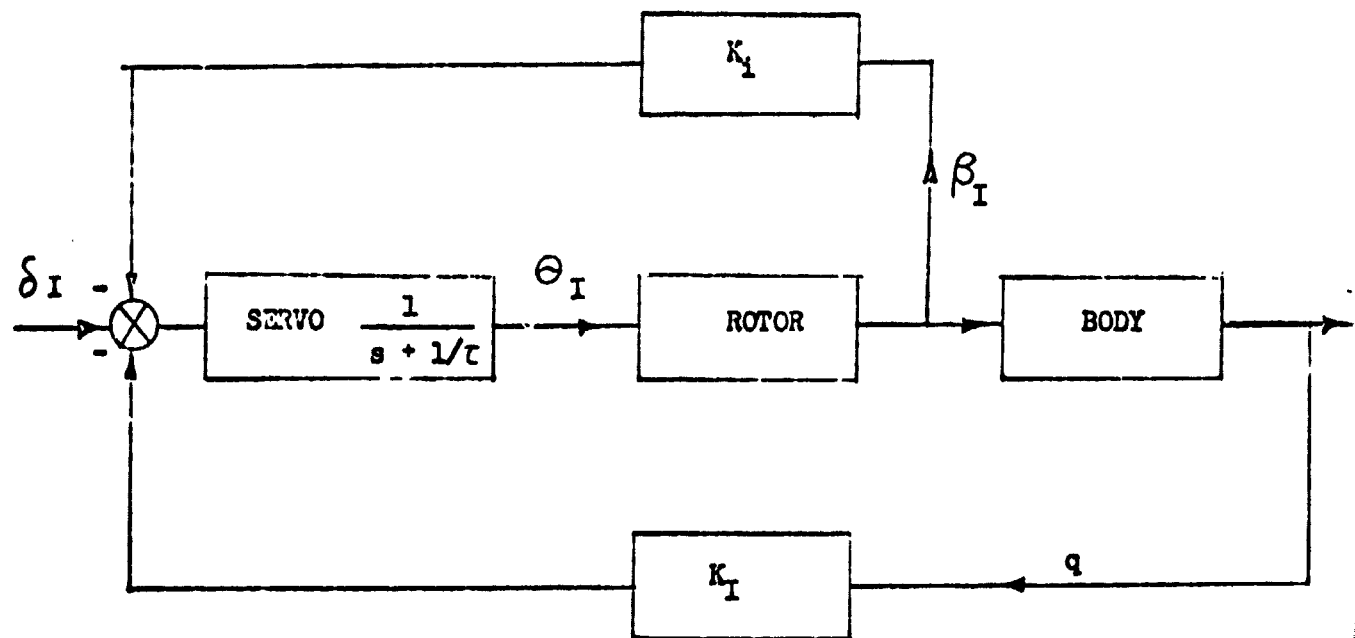
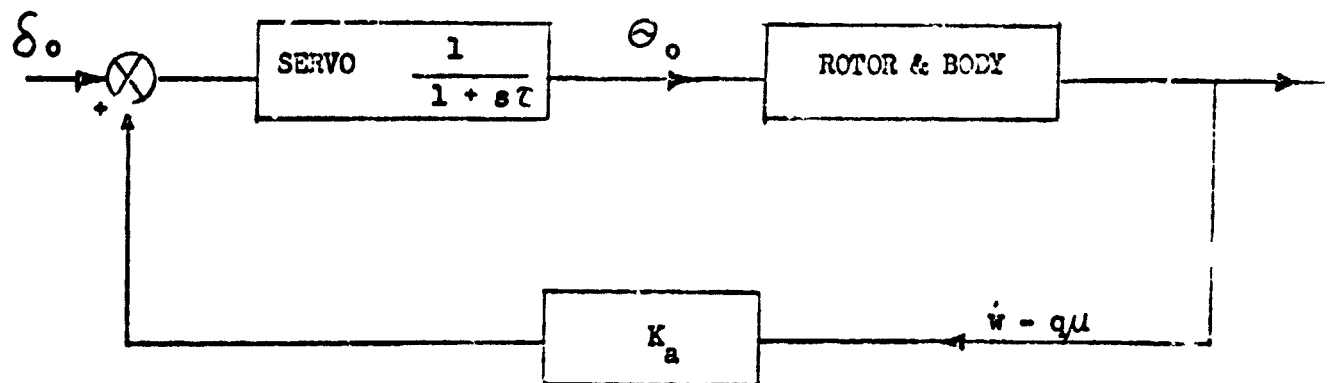
ROTOR TILTING FEEDBACK SYSTEMNORMAL ACCELERATION FEEDBACK SYSTEM

Fig. 1

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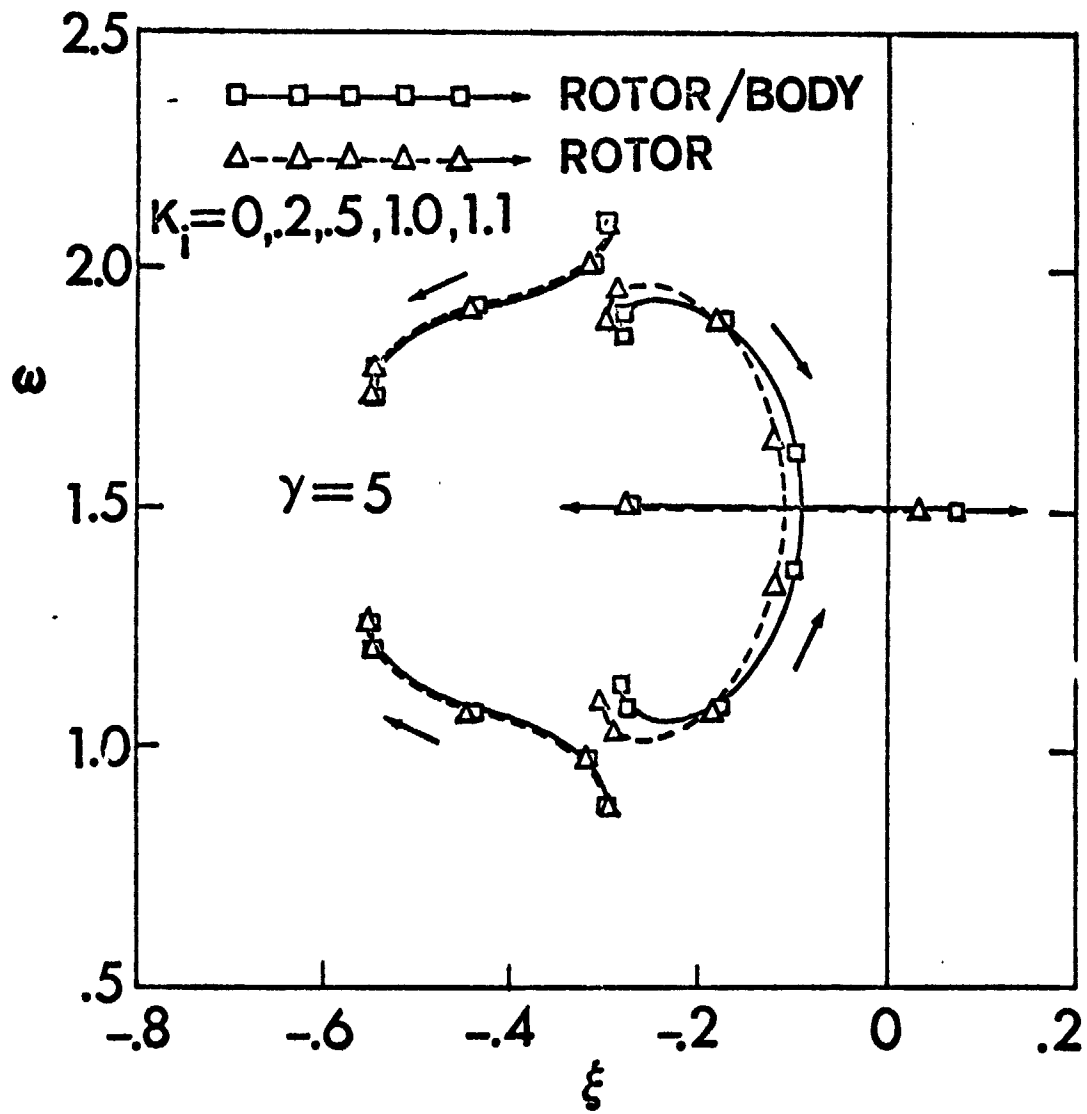


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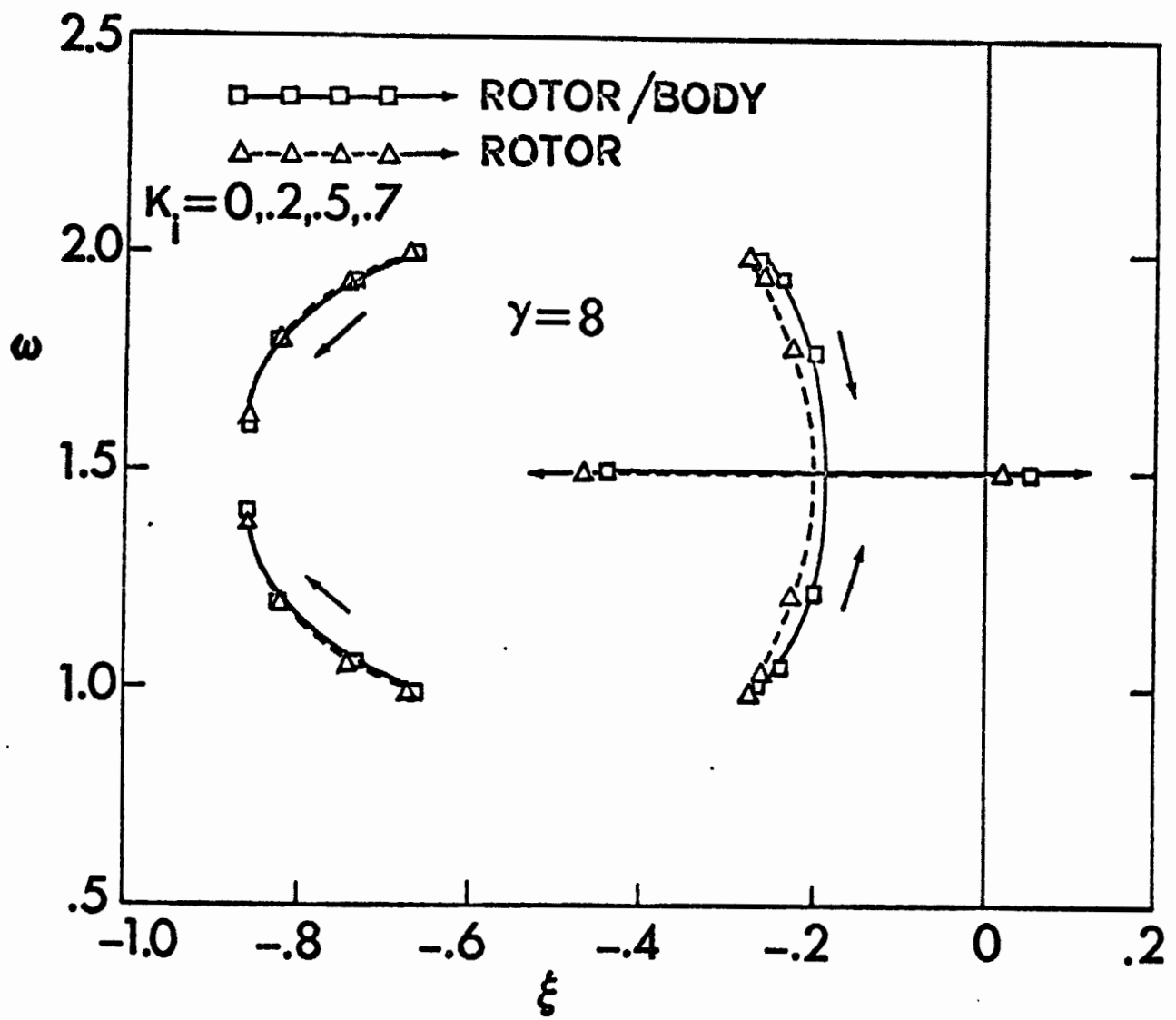


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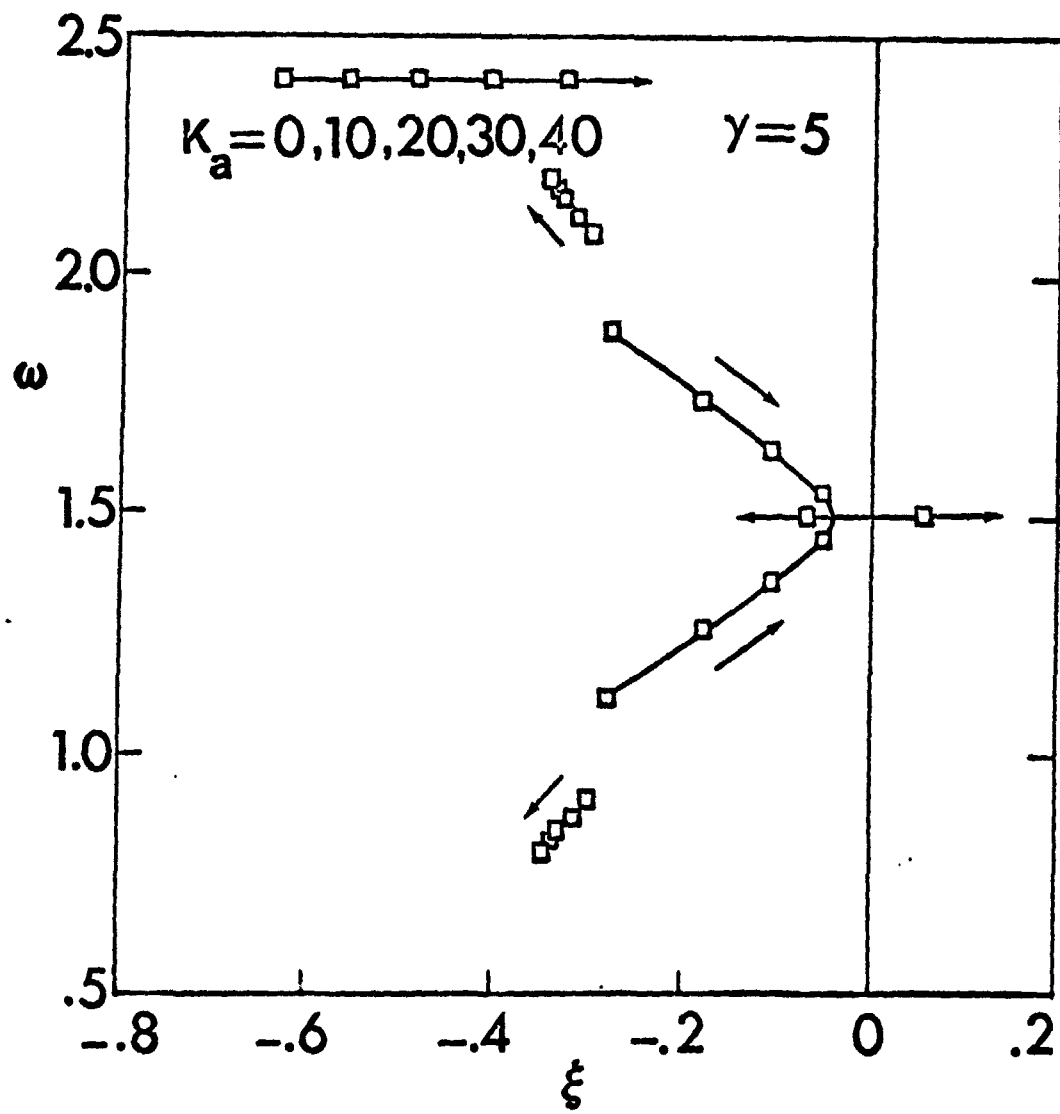


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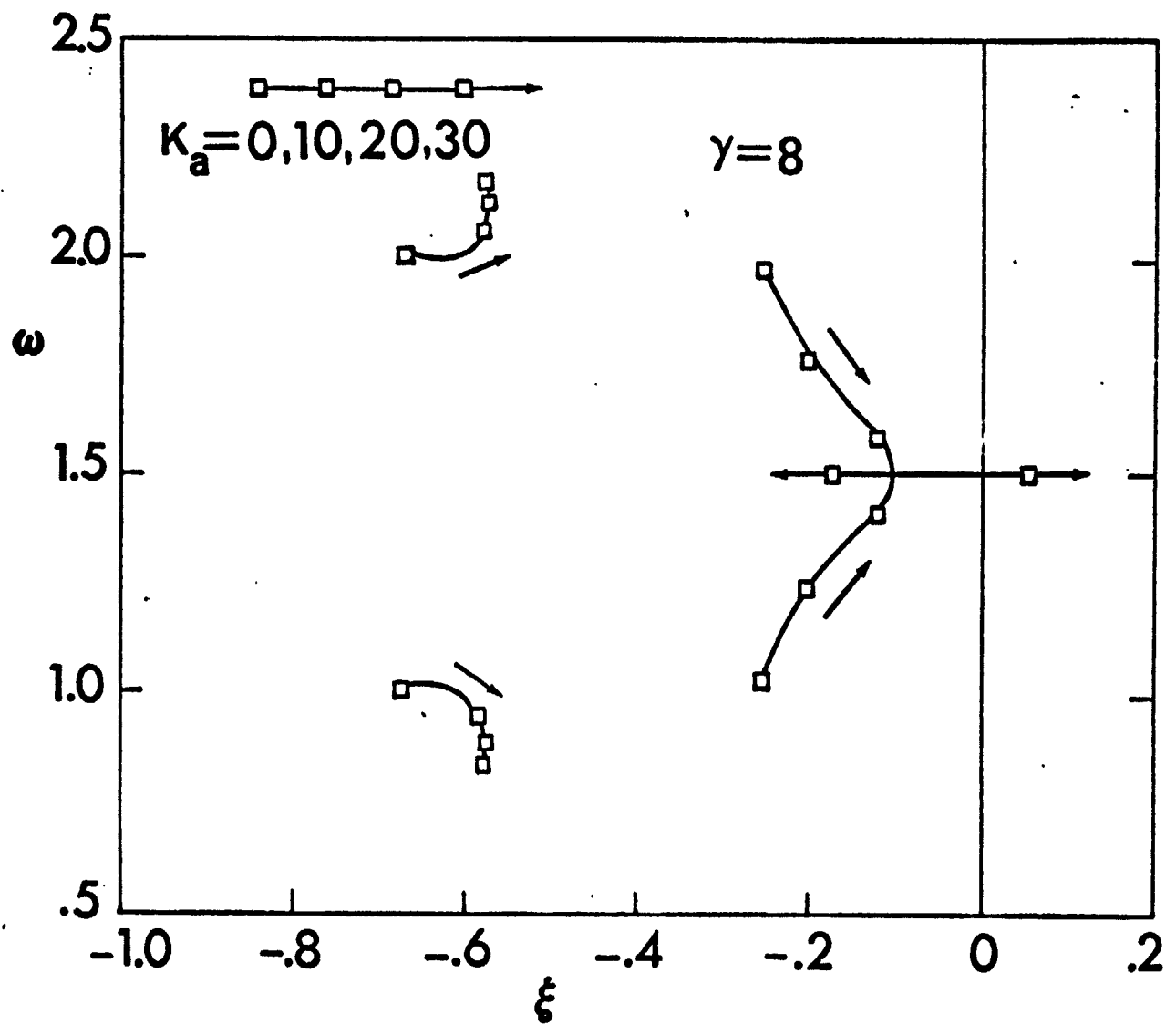


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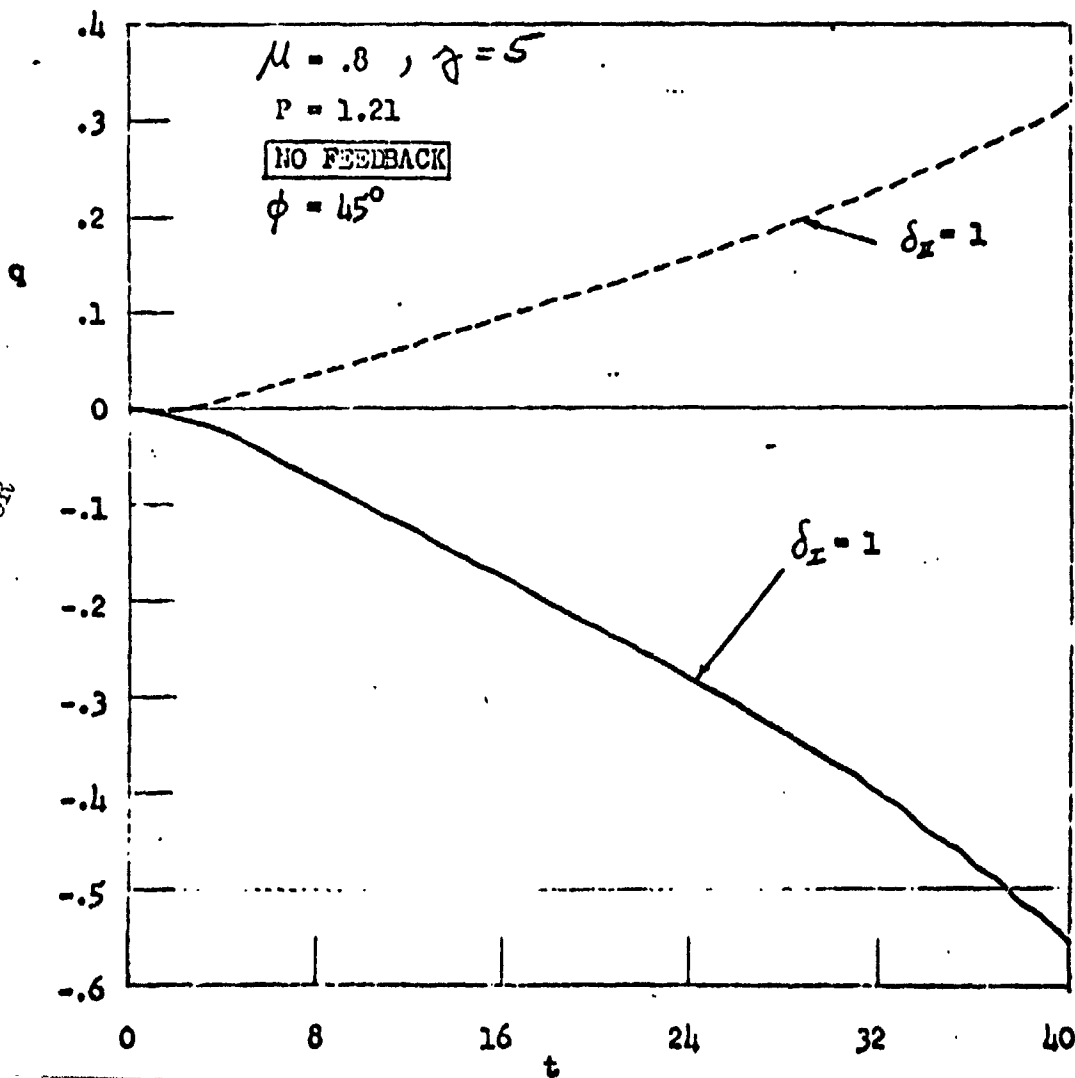
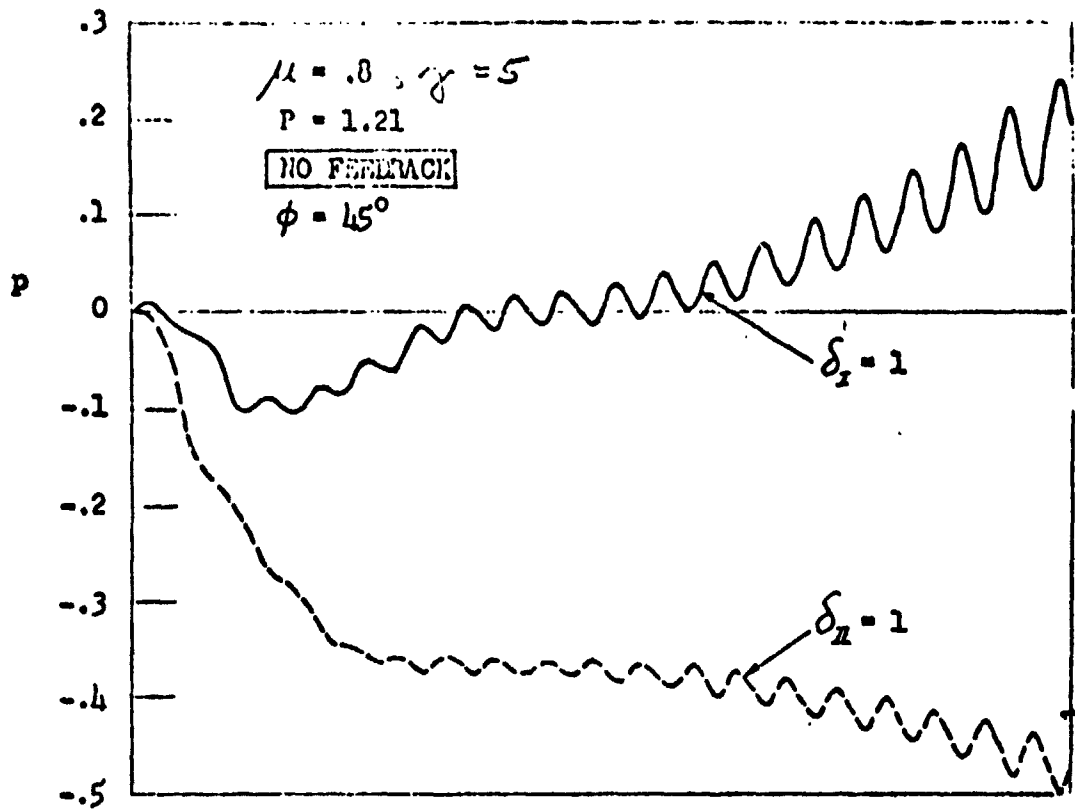


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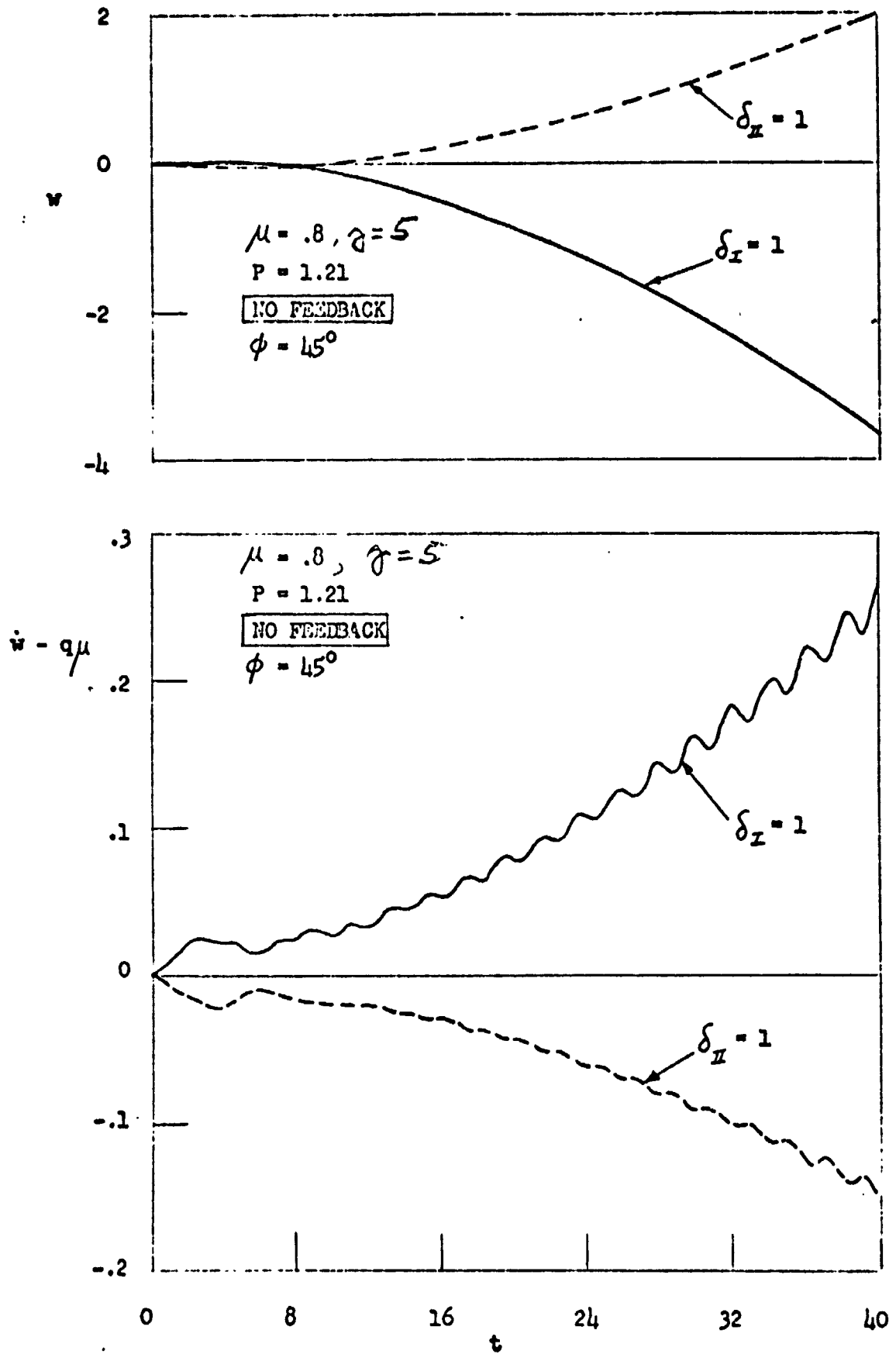


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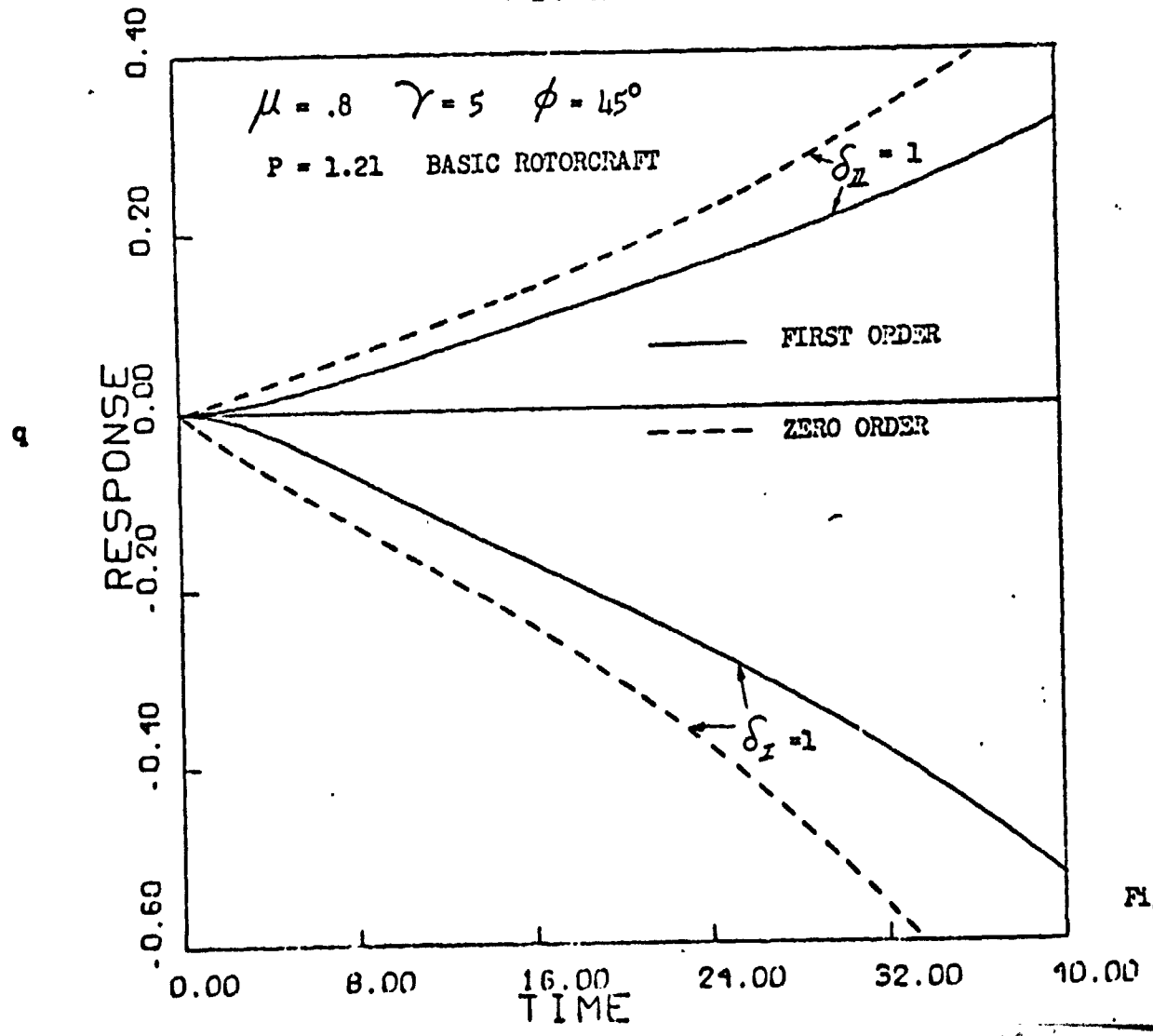
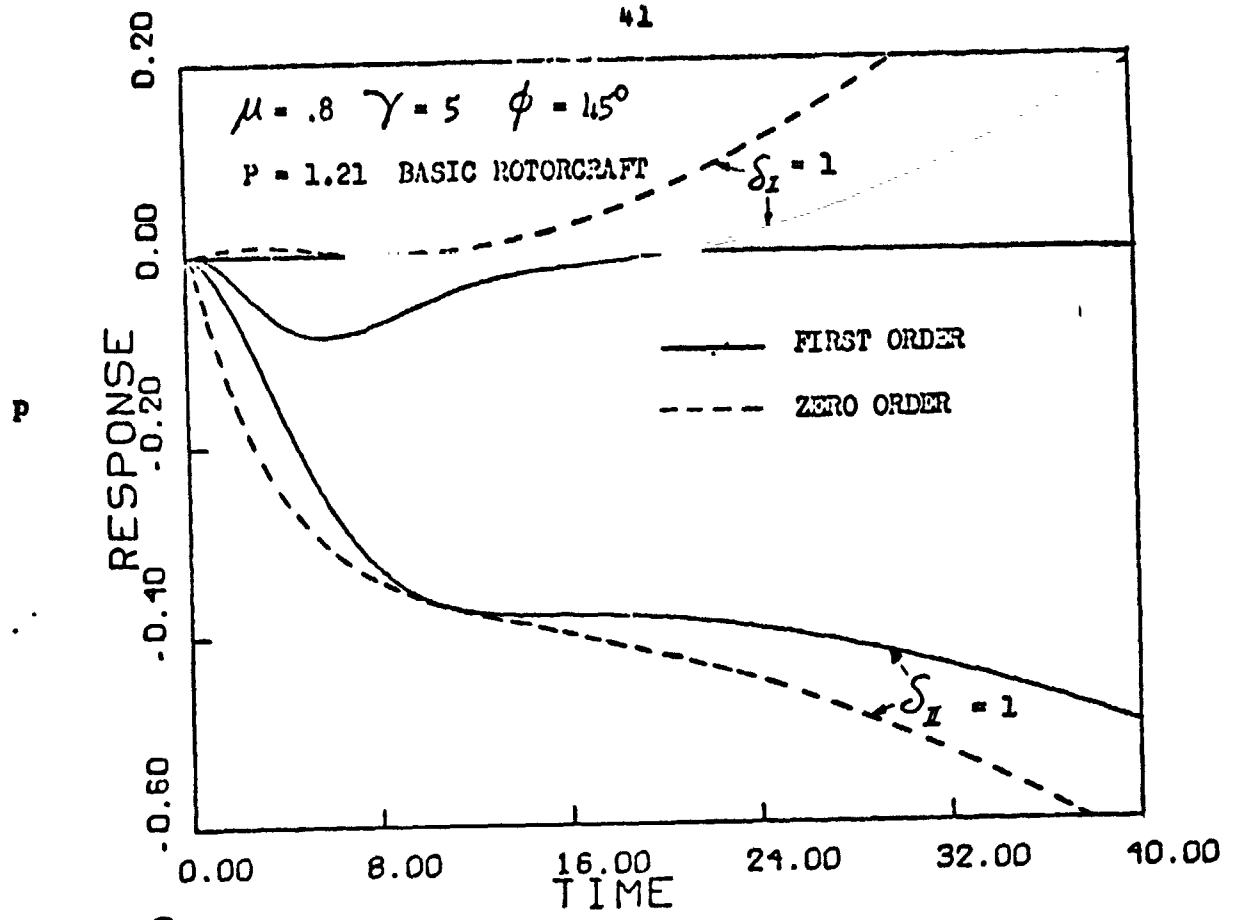


FIG. 5

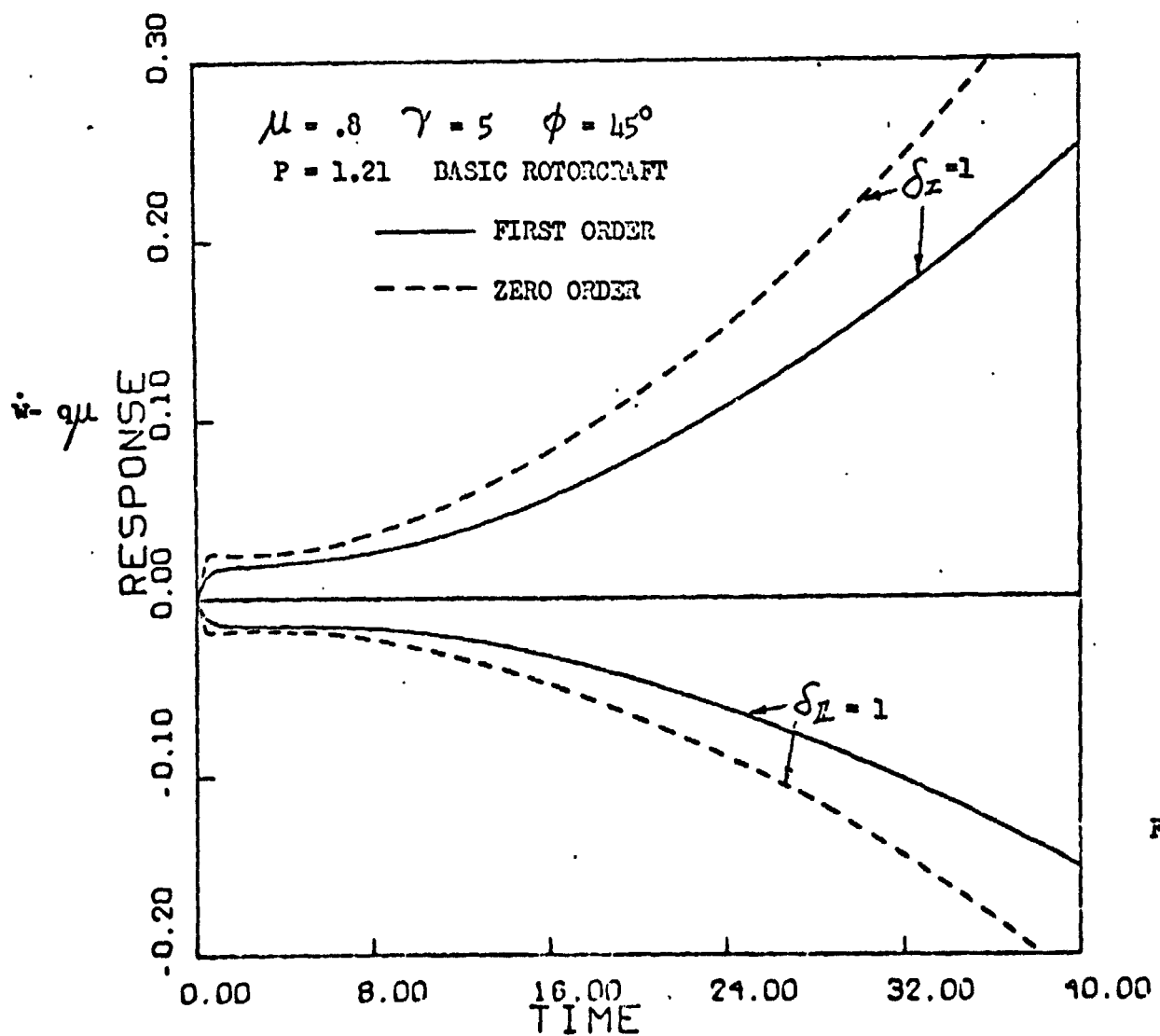
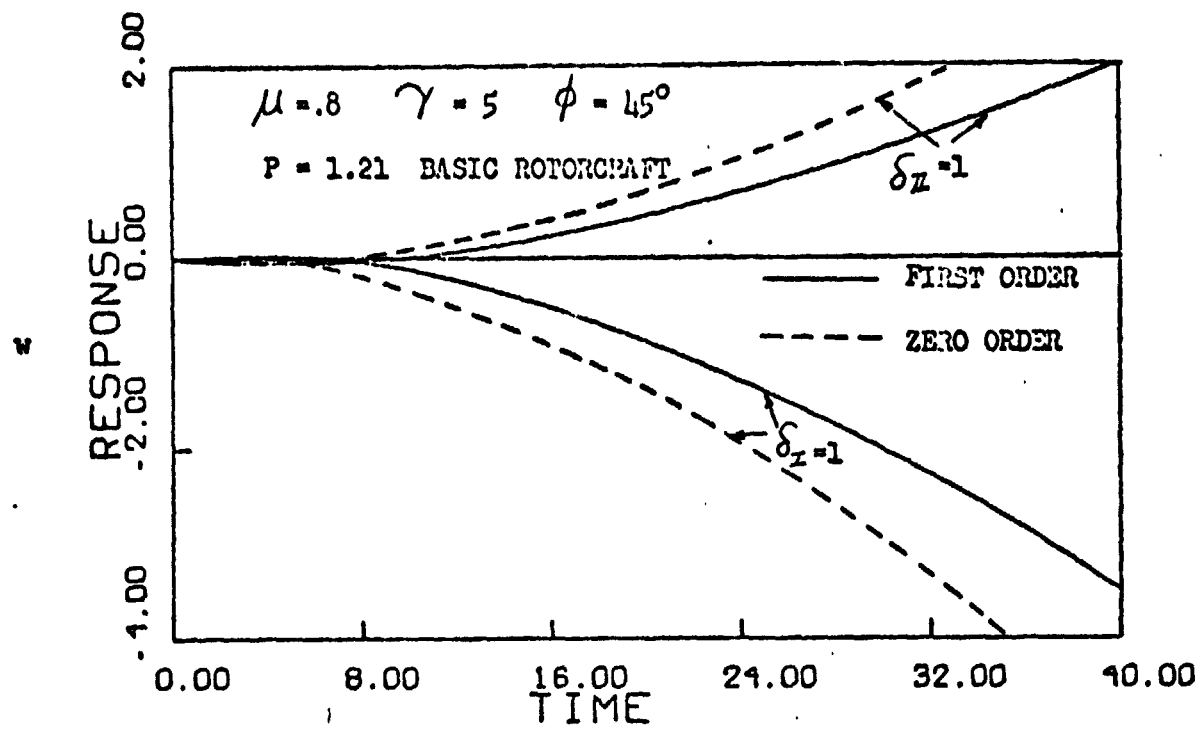


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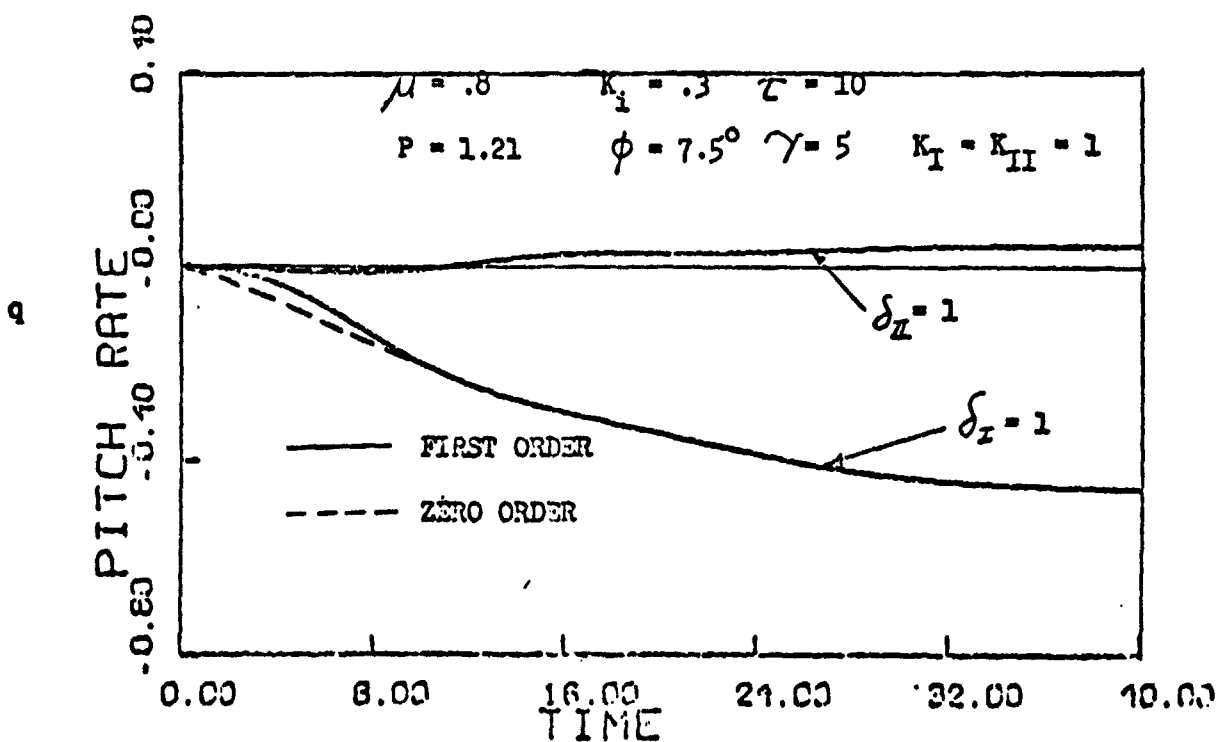
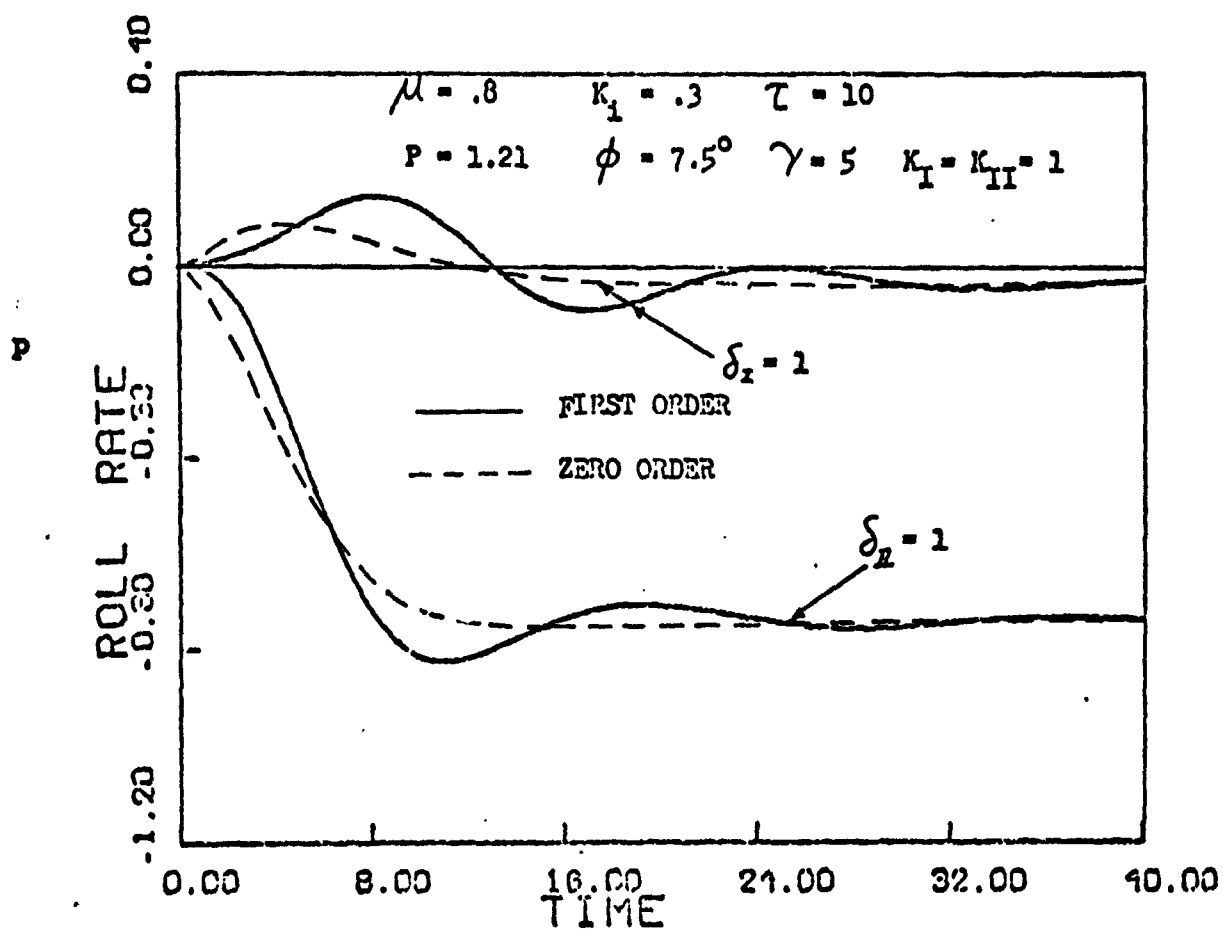


FIG. 6

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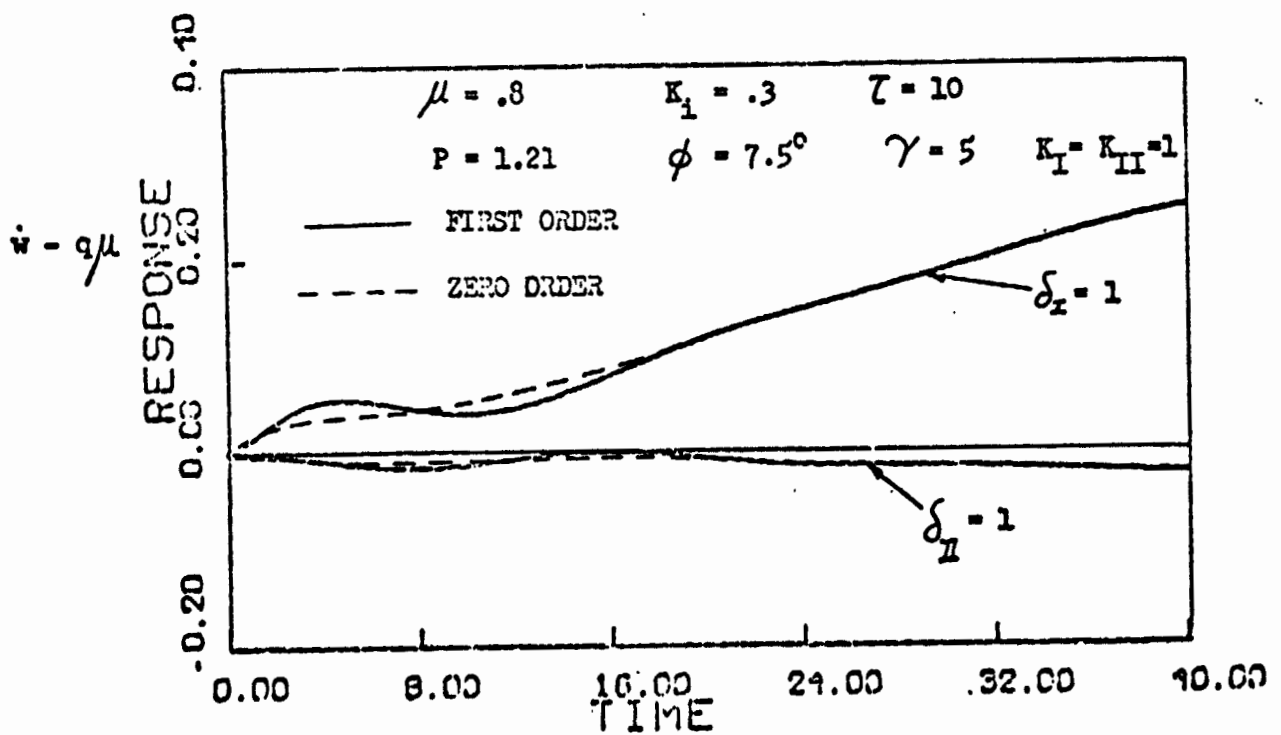
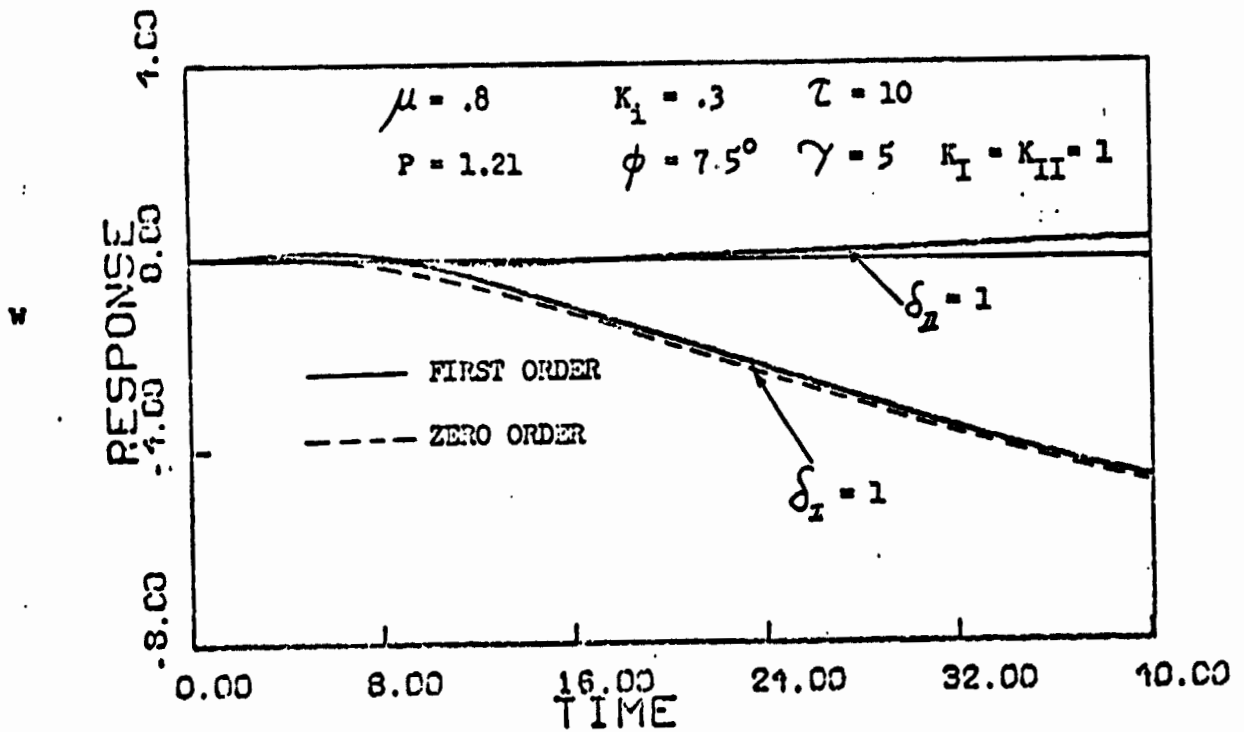


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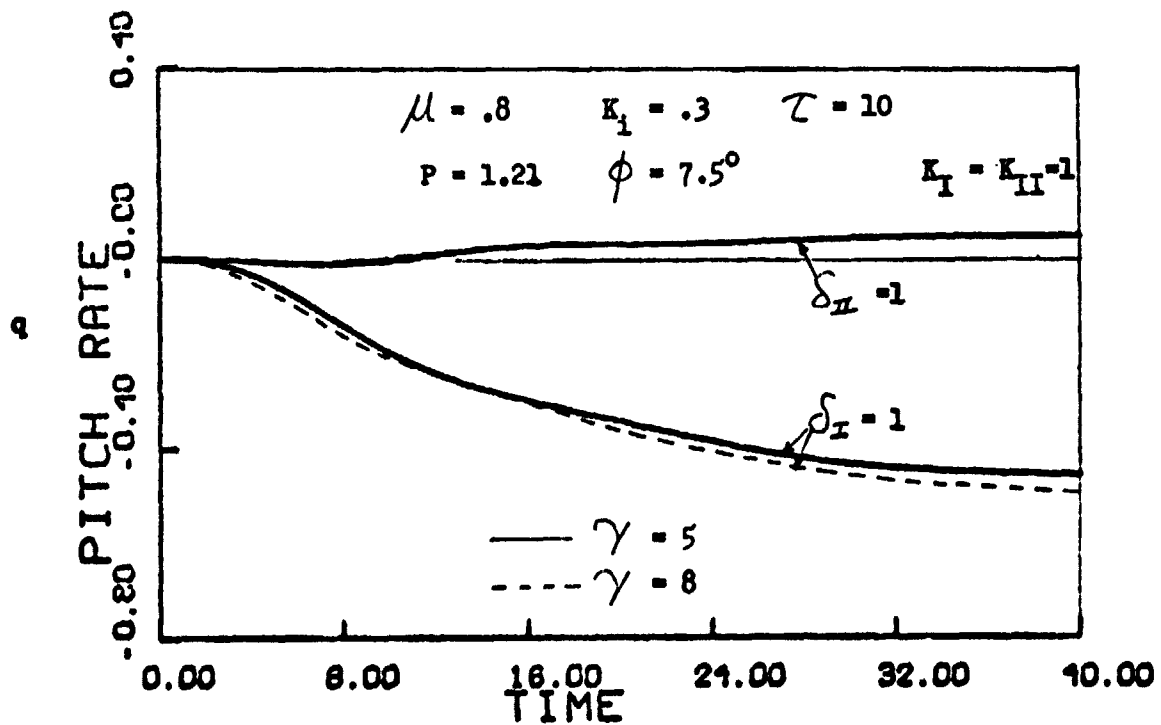
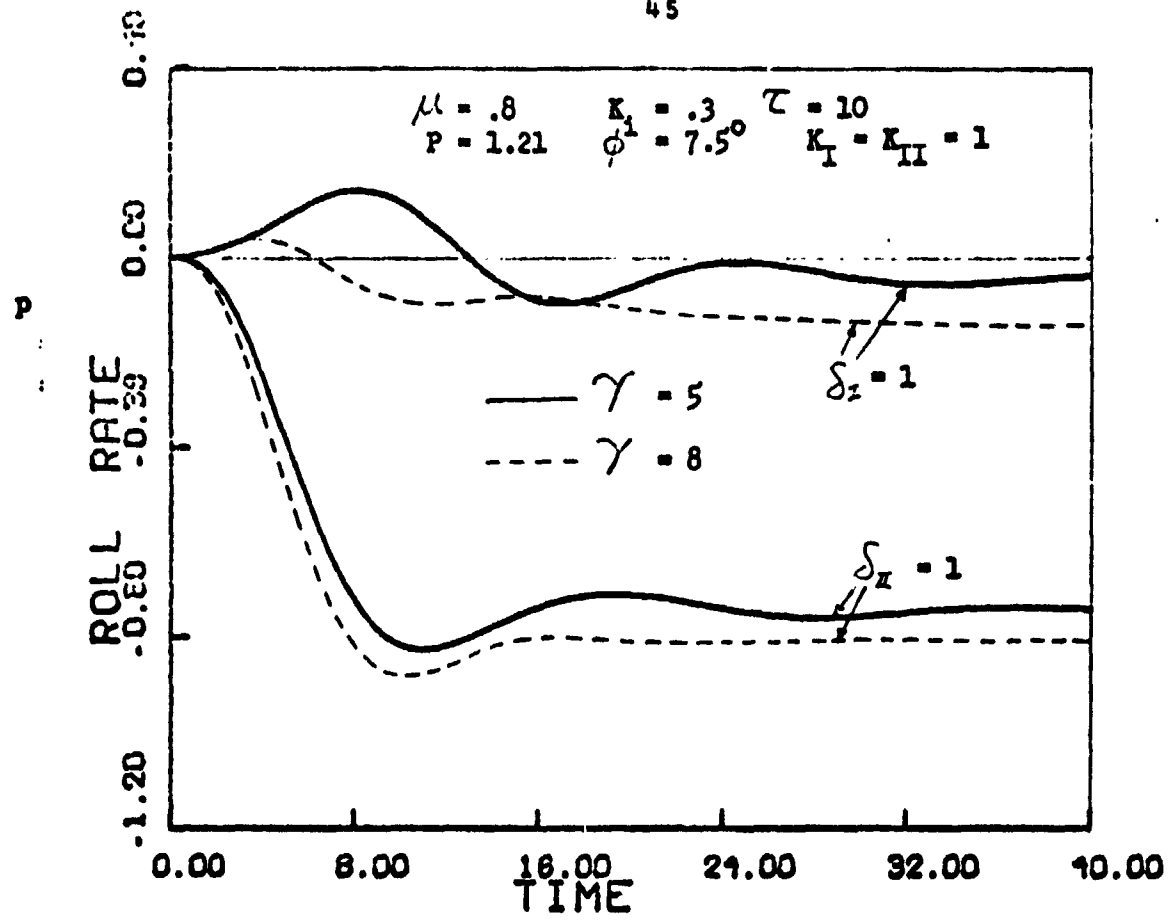


Fig. 7

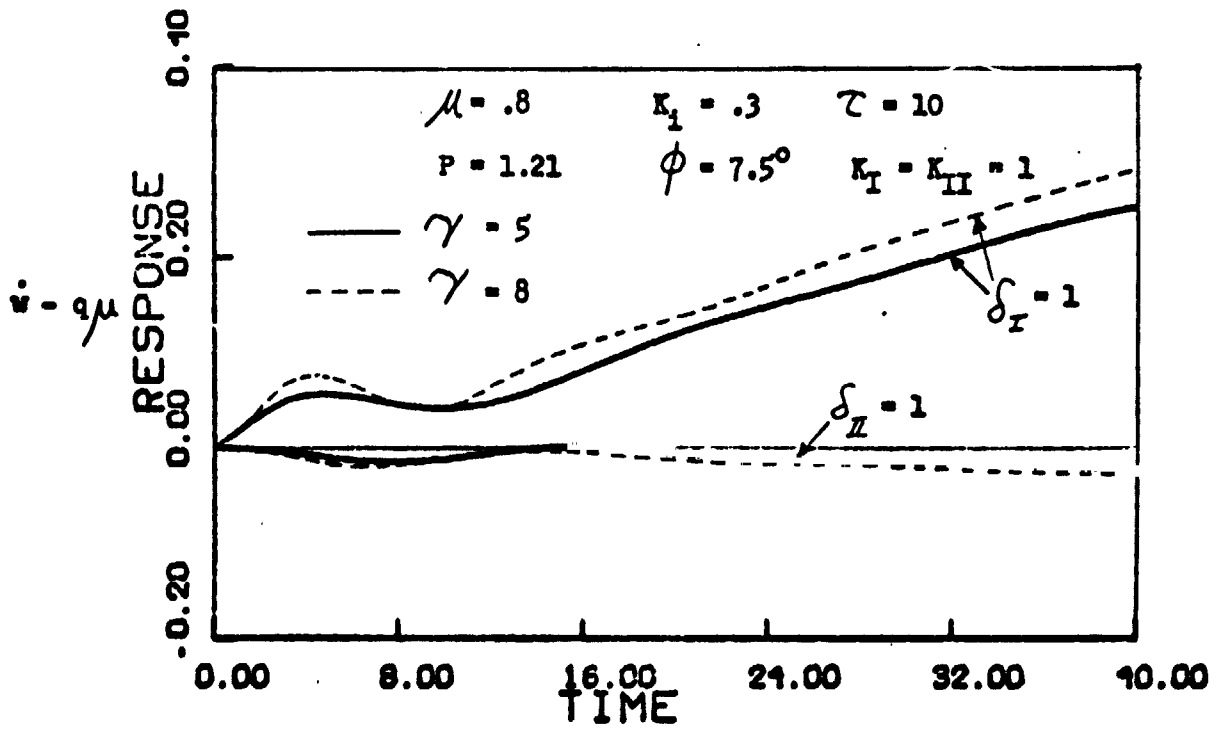
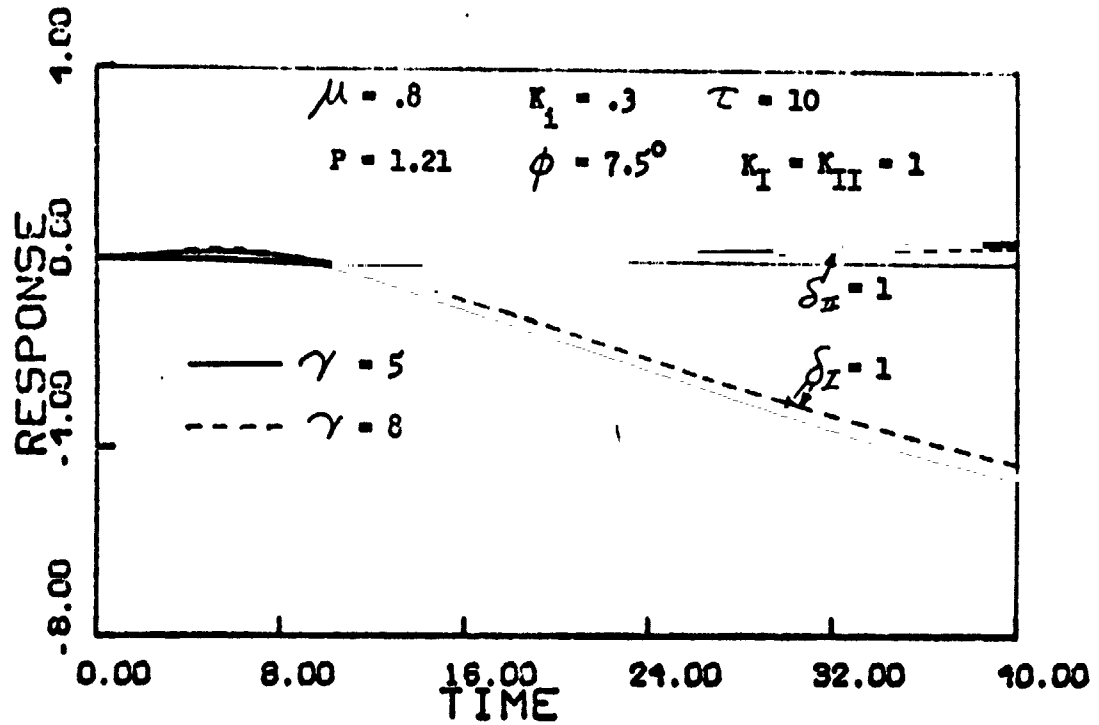


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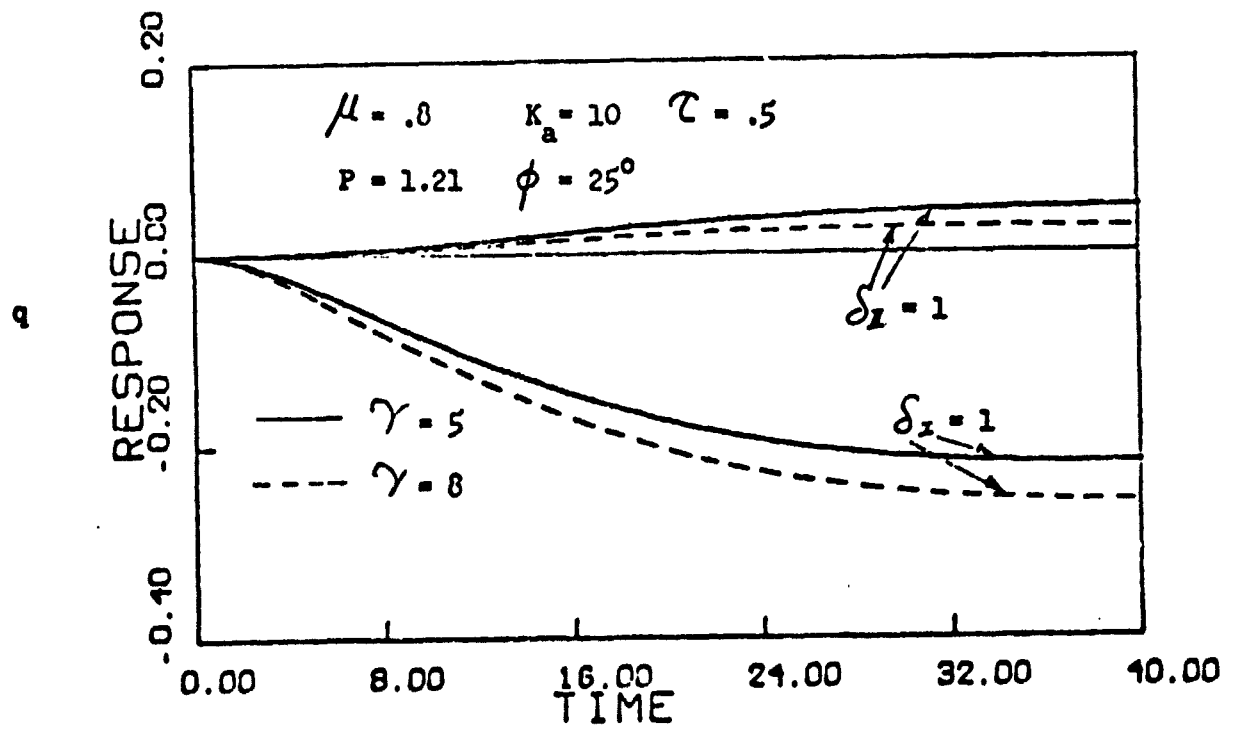
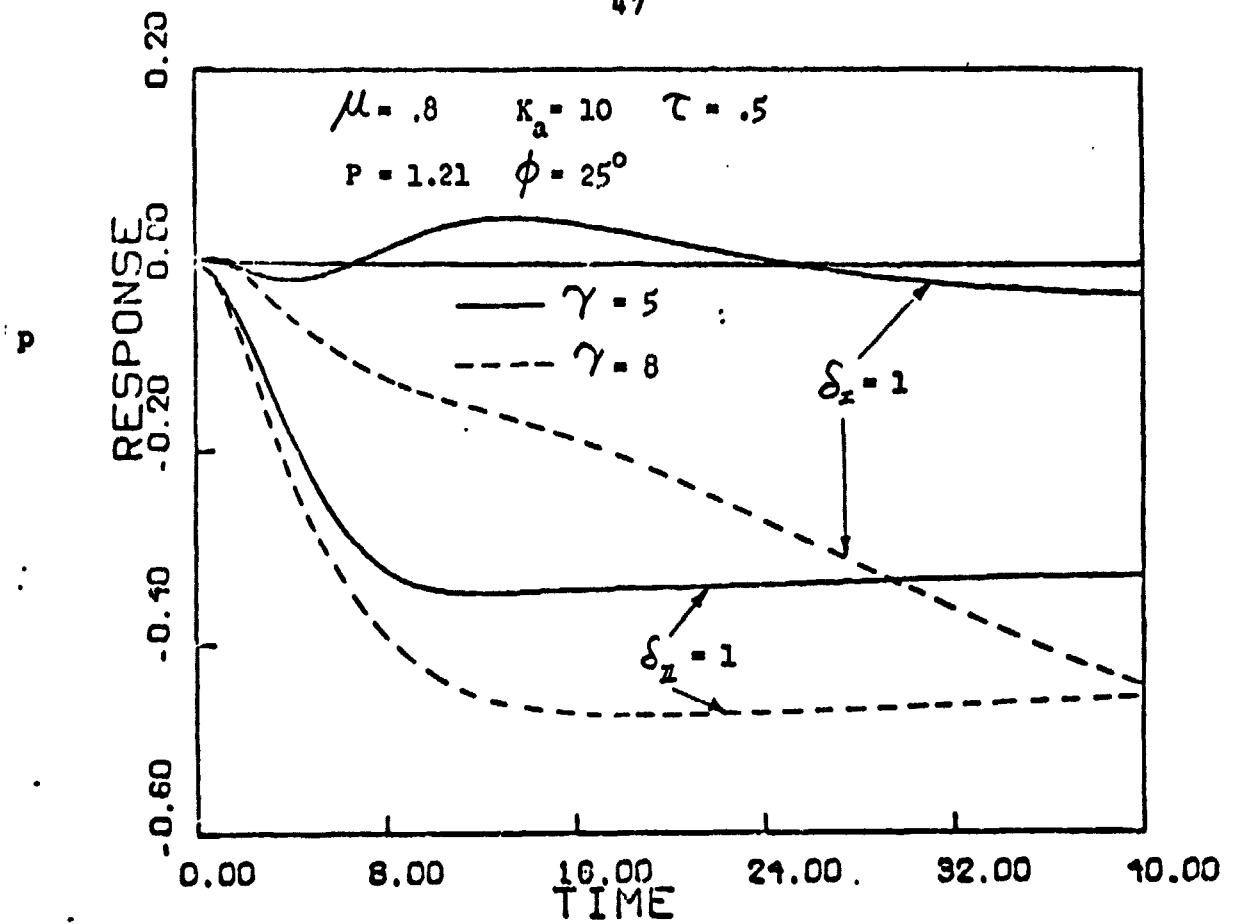


FIG. 8

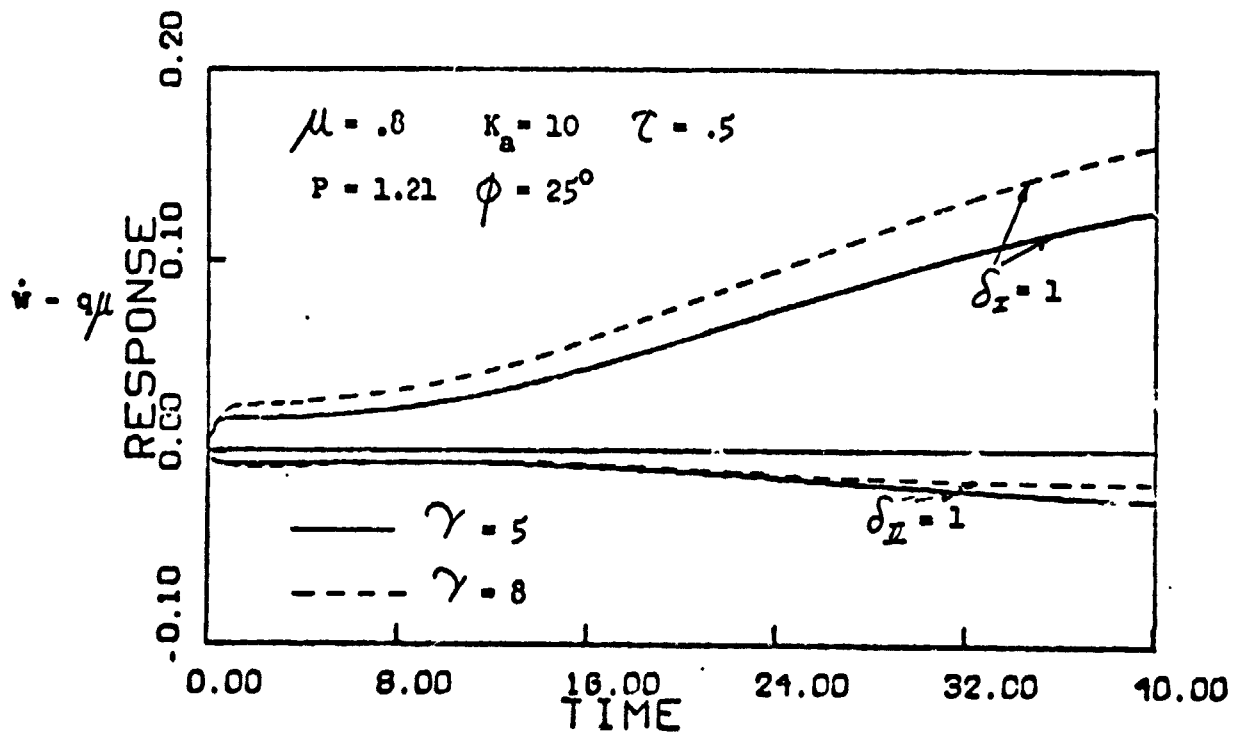
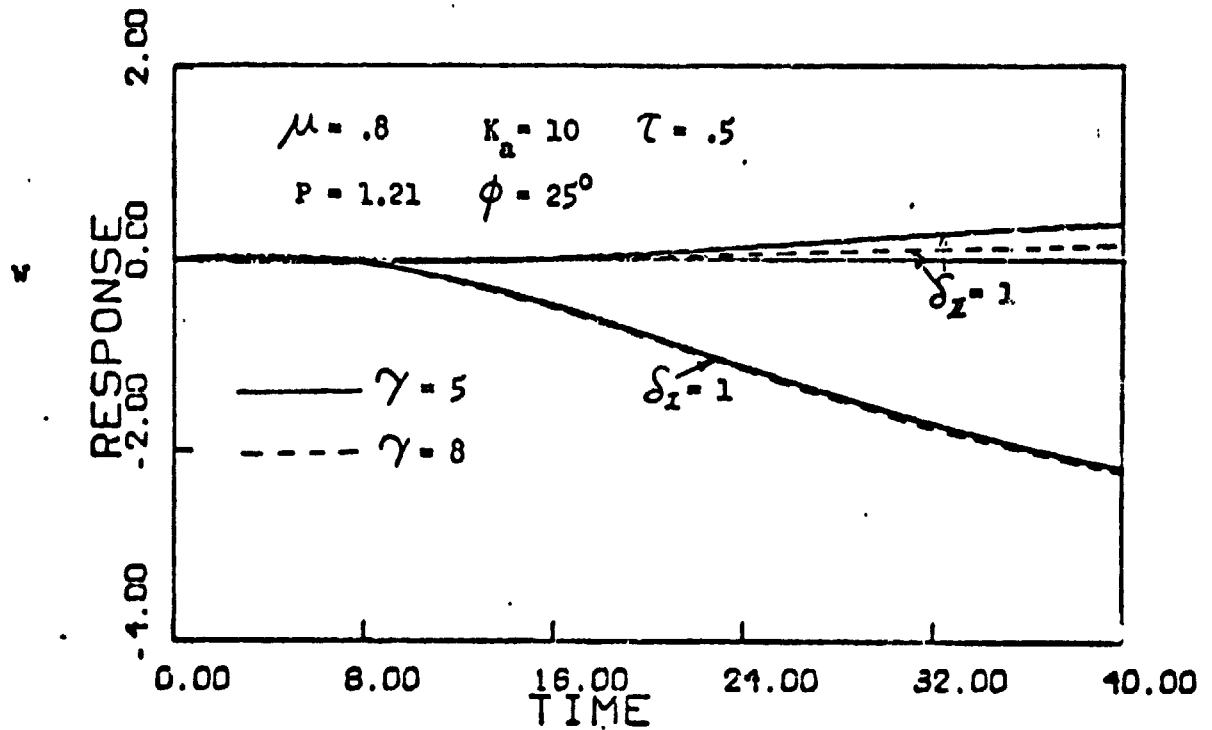


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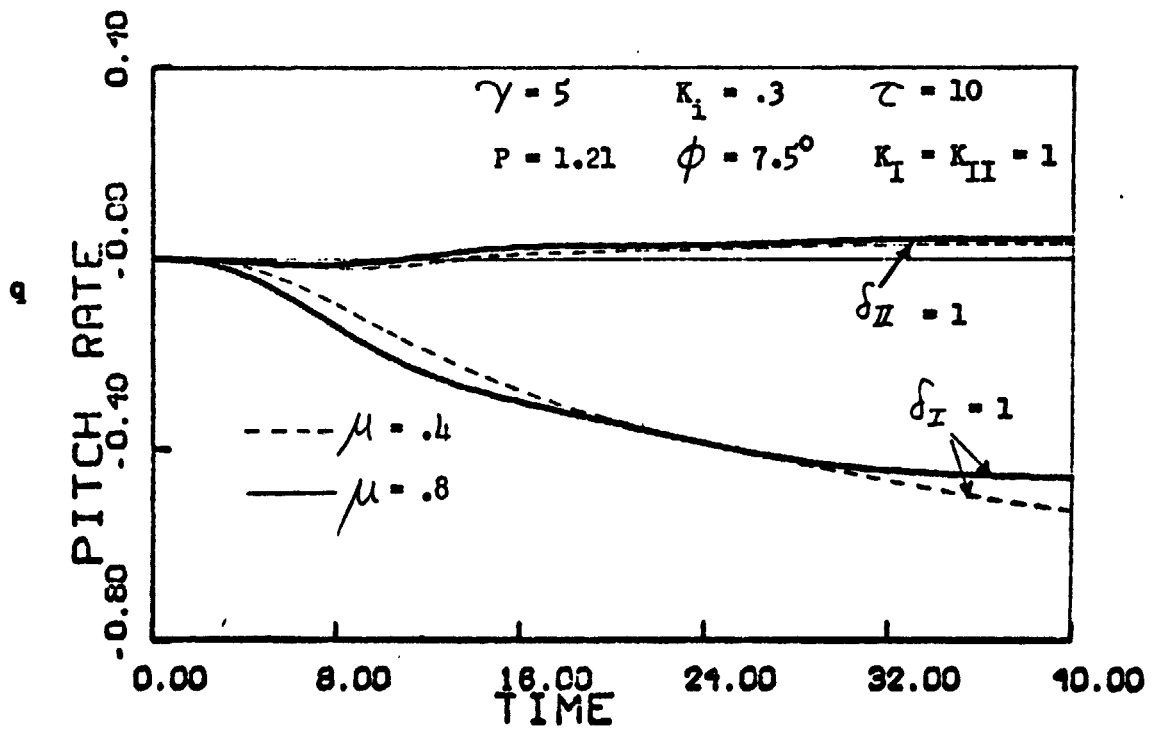
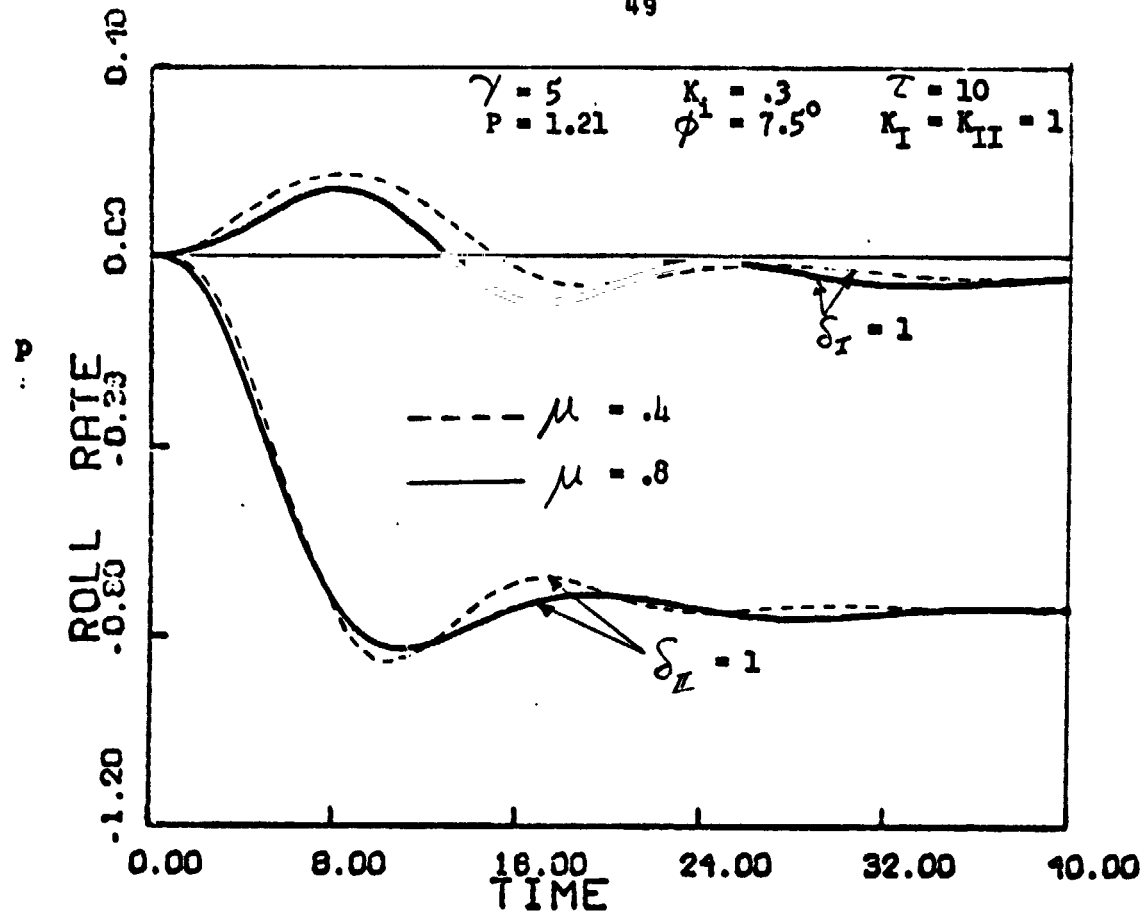


Fig. 9

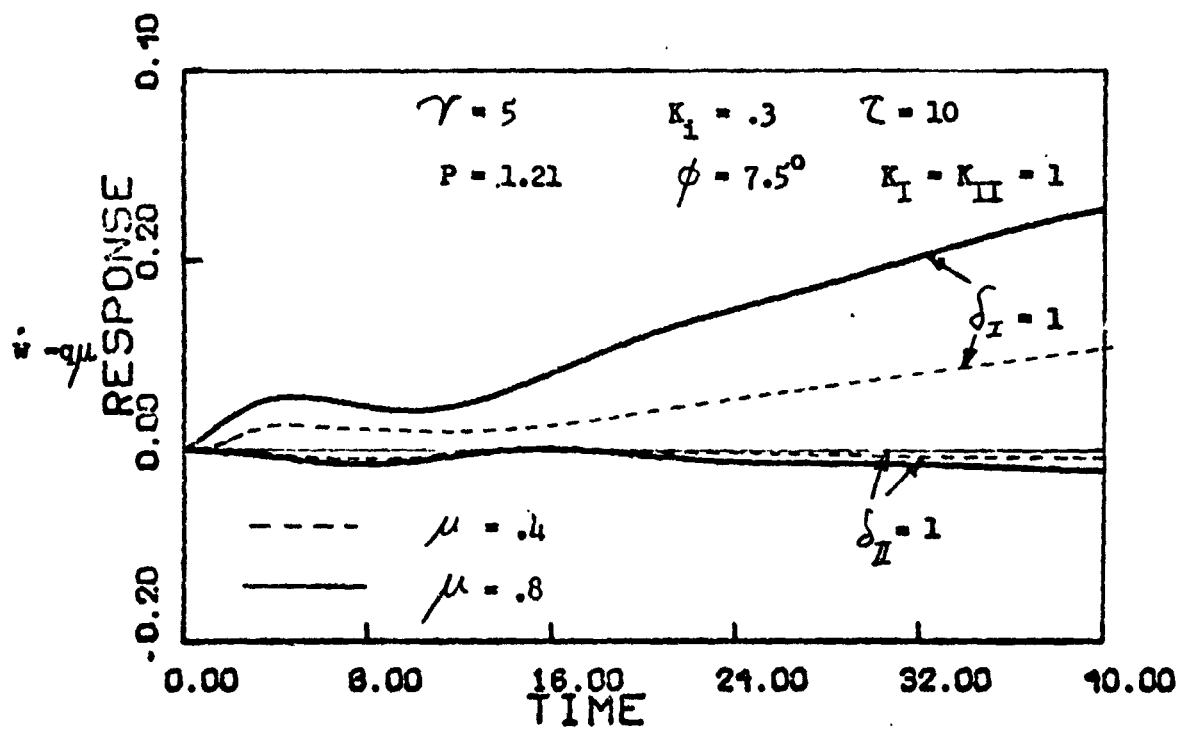
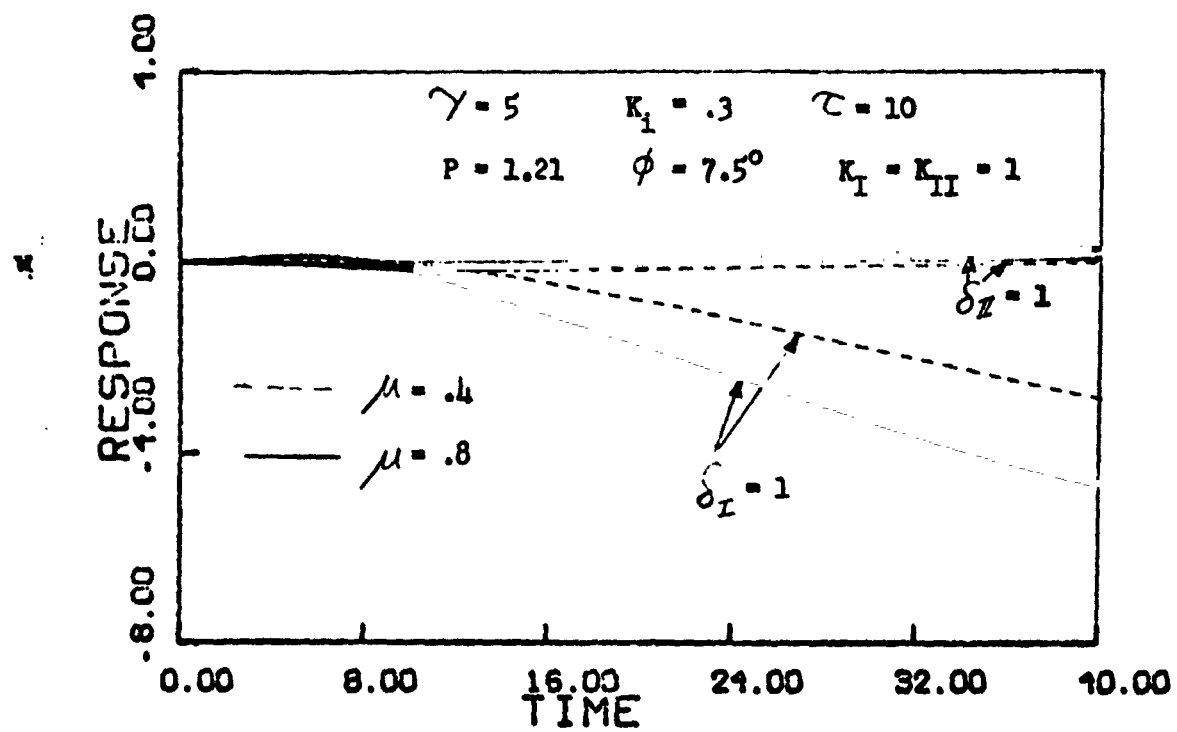


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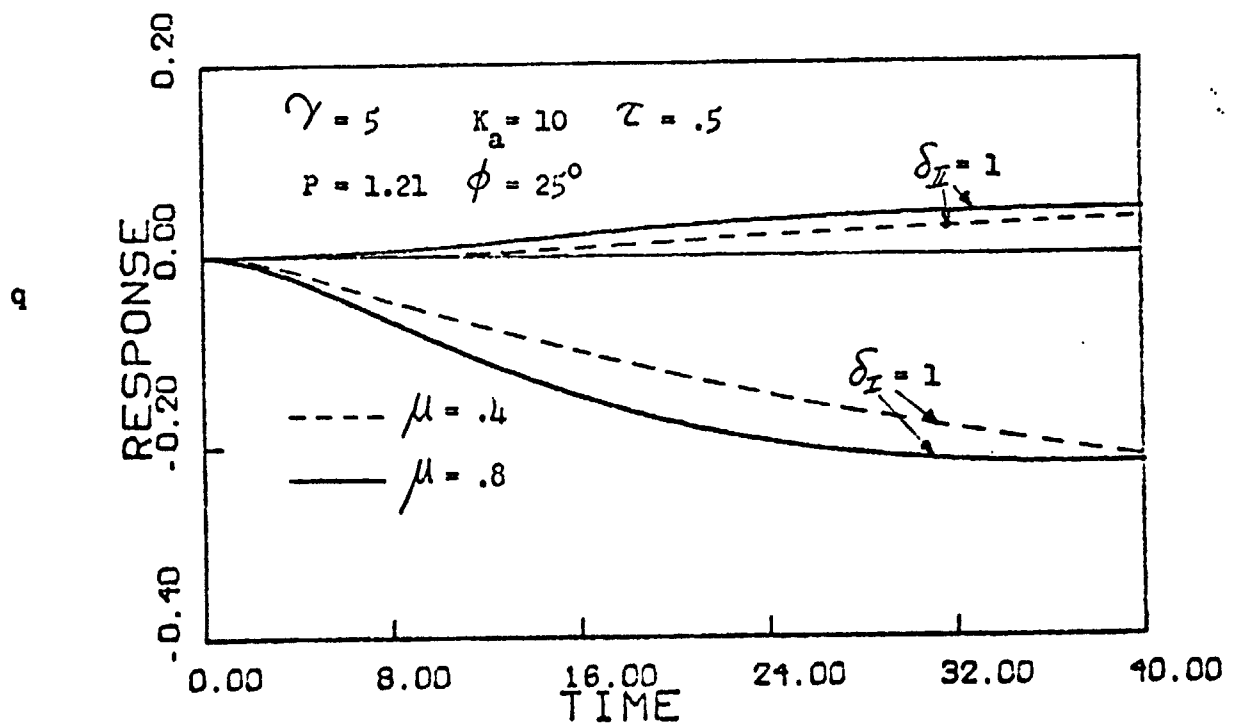
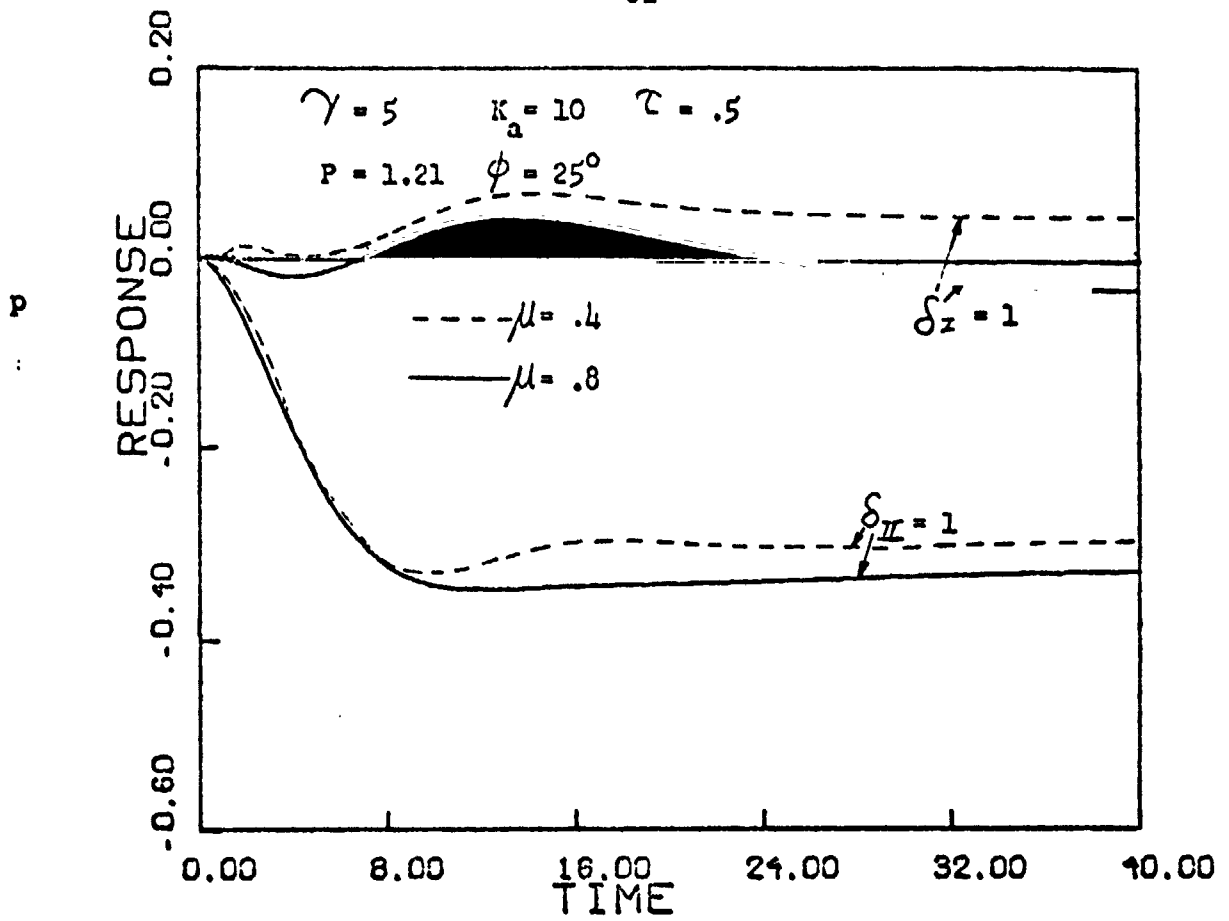


Fig. 10

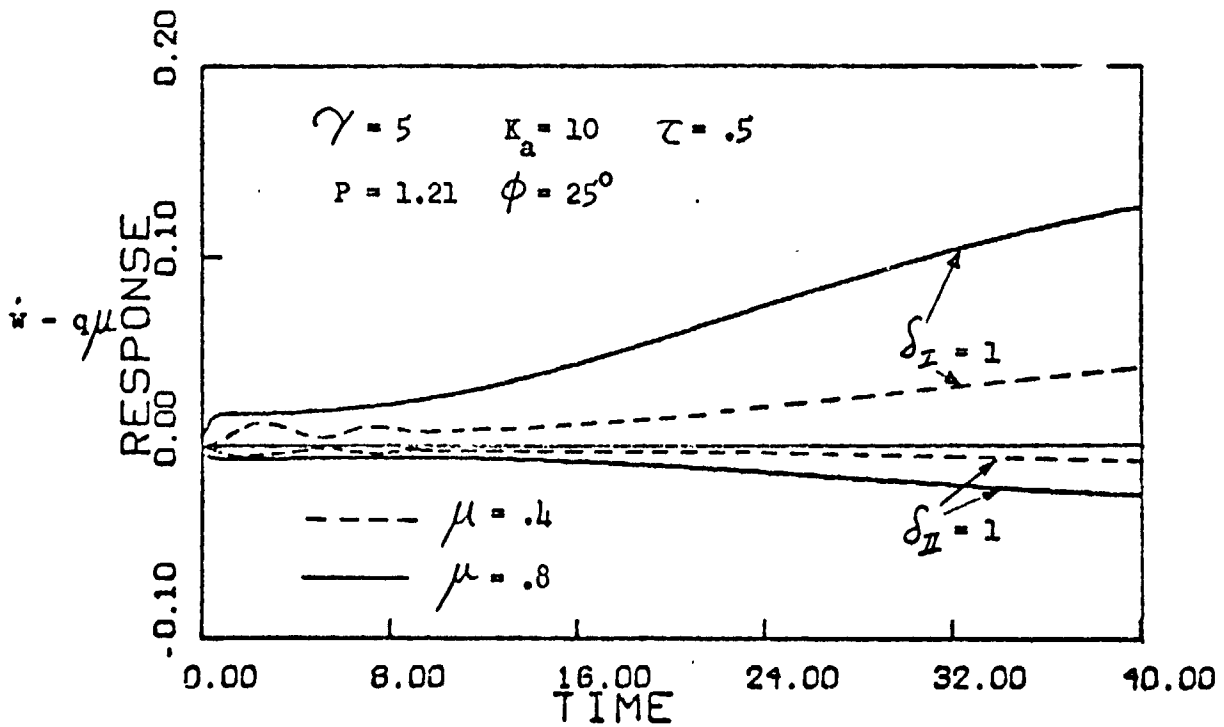
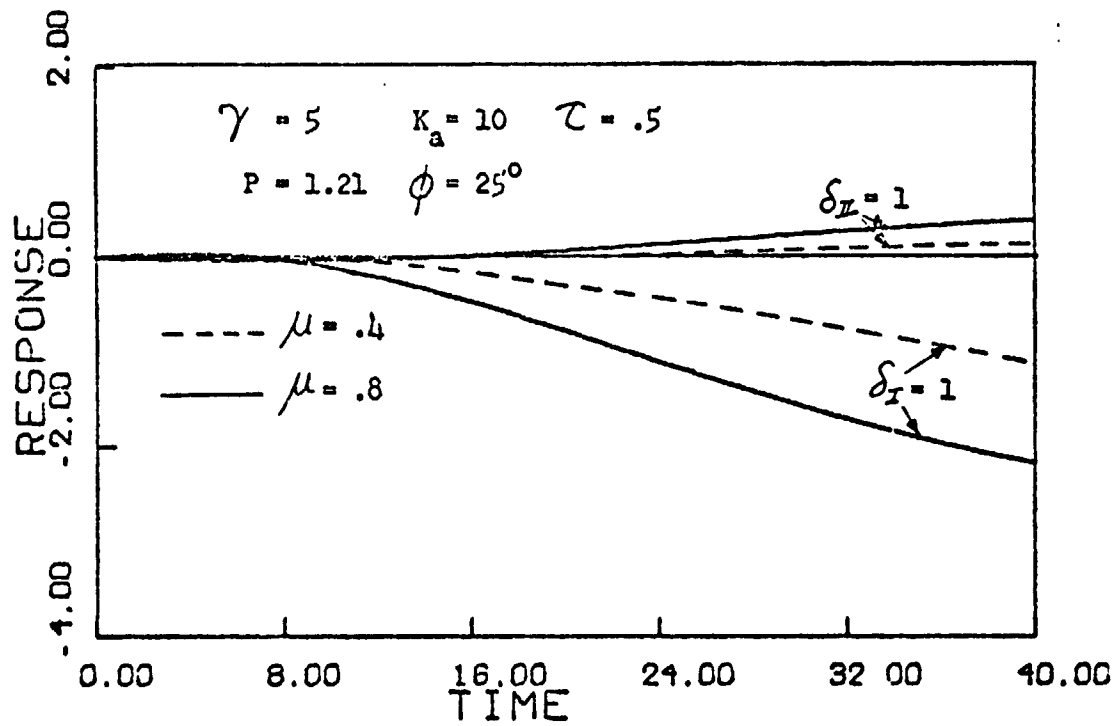


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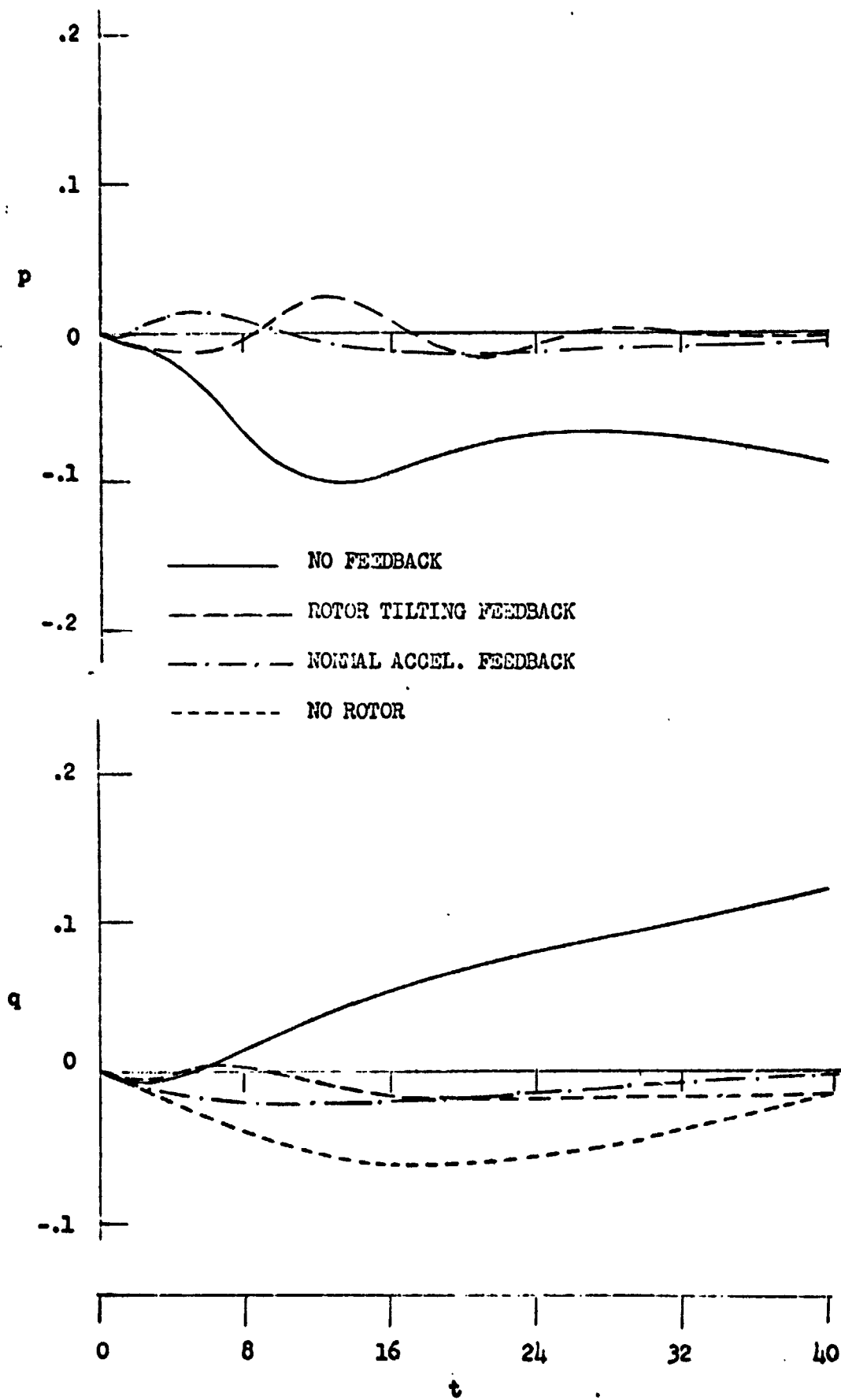


Fig. 11

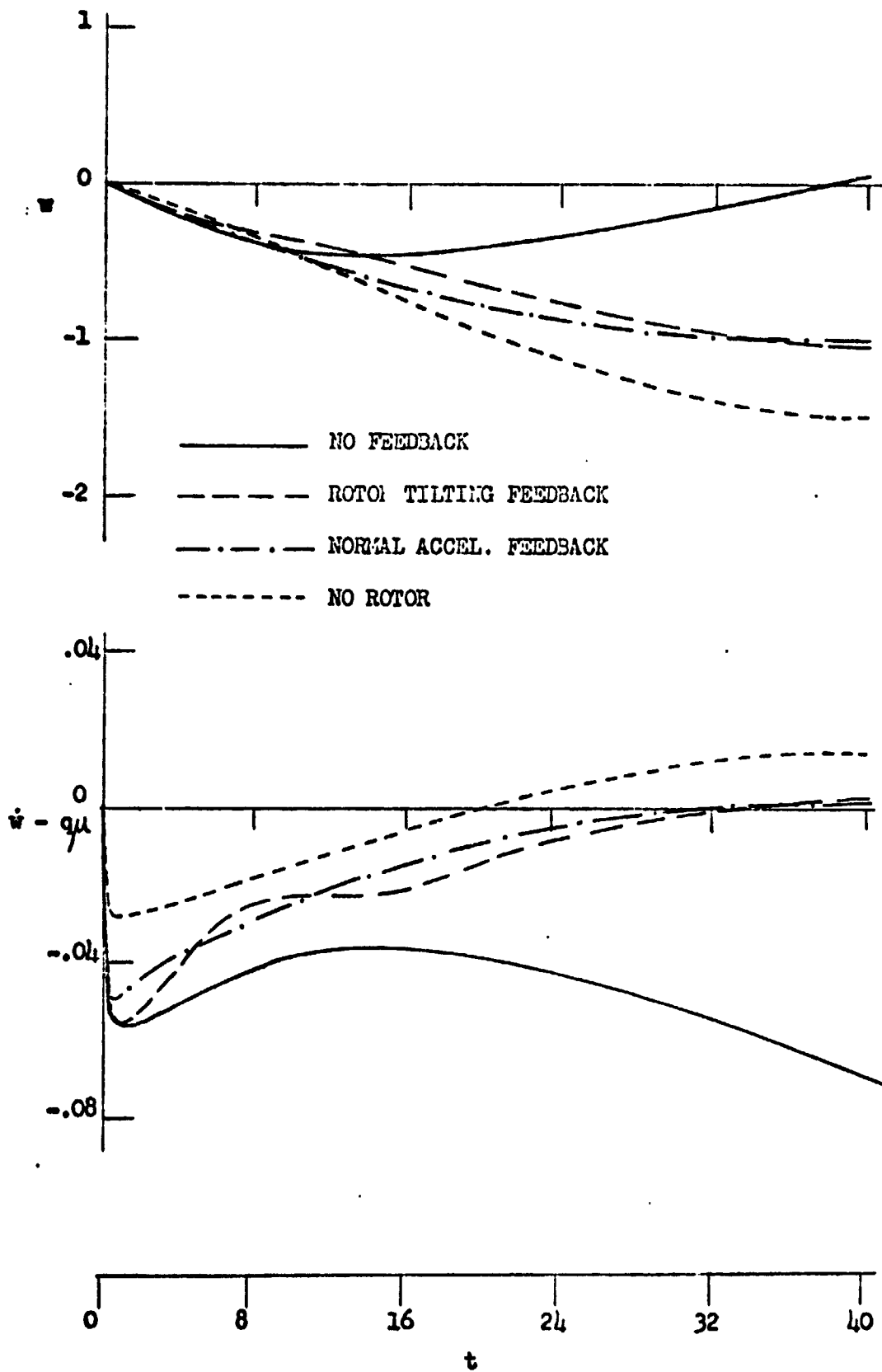


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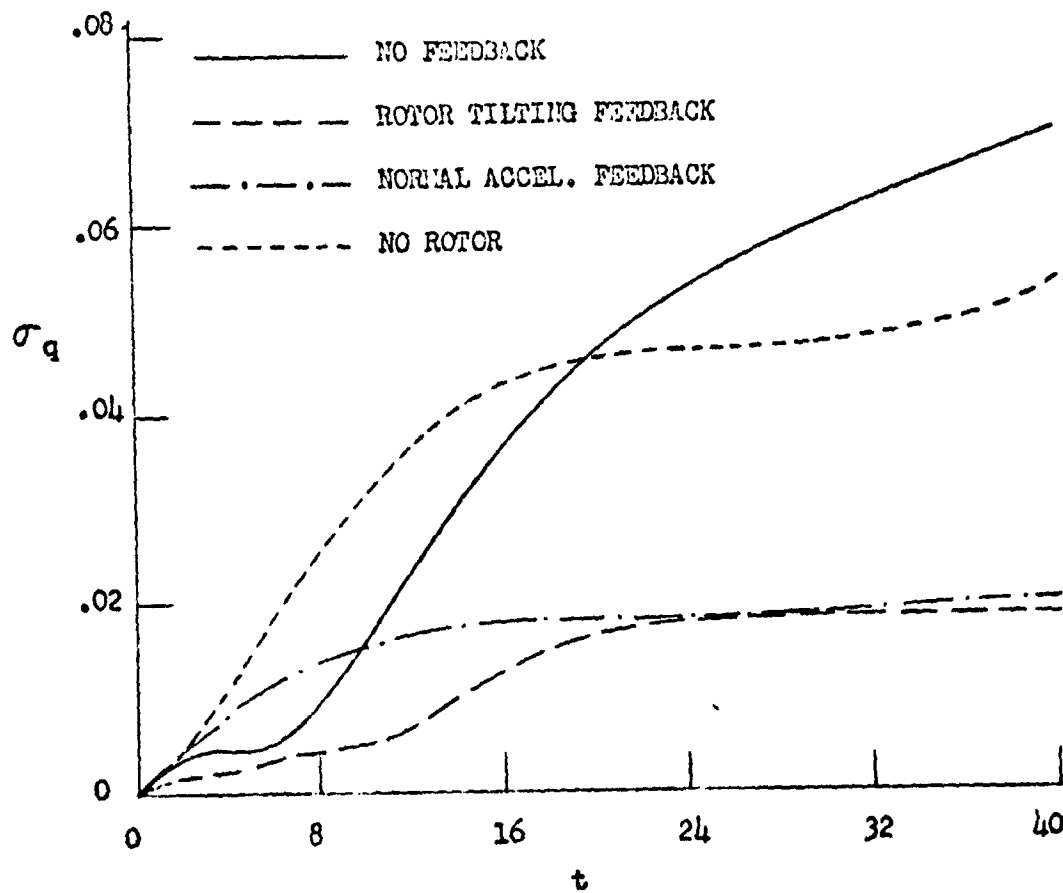
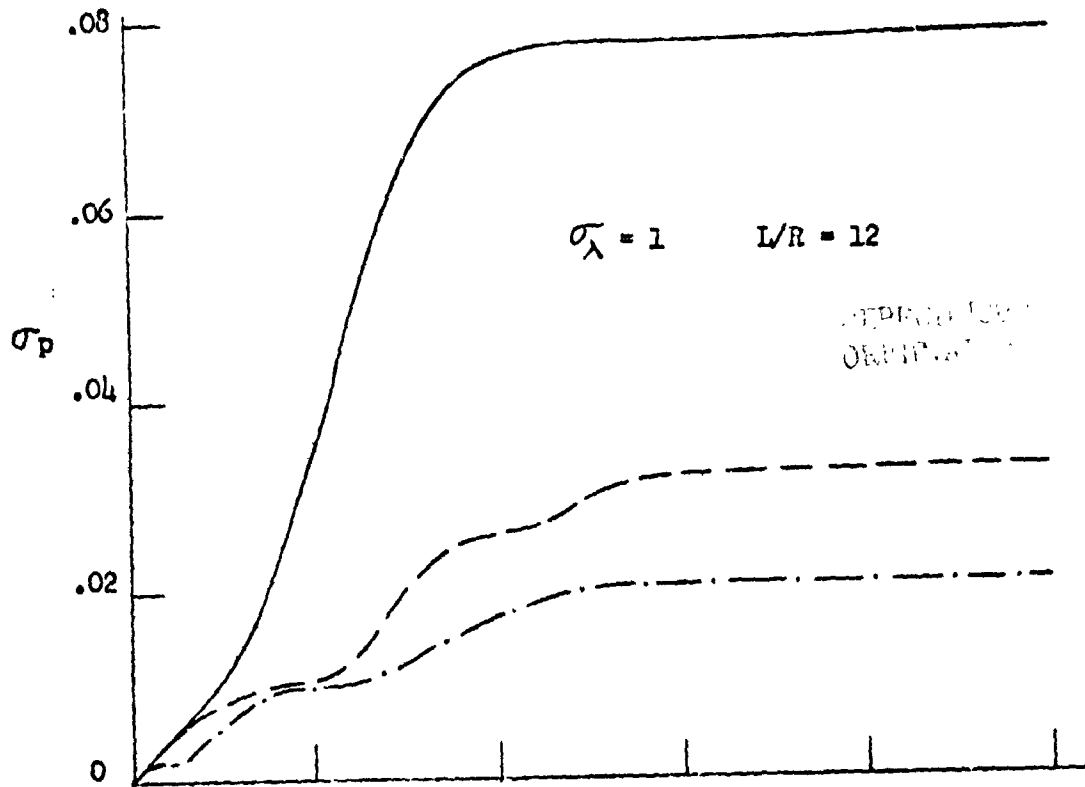


Fig. 12

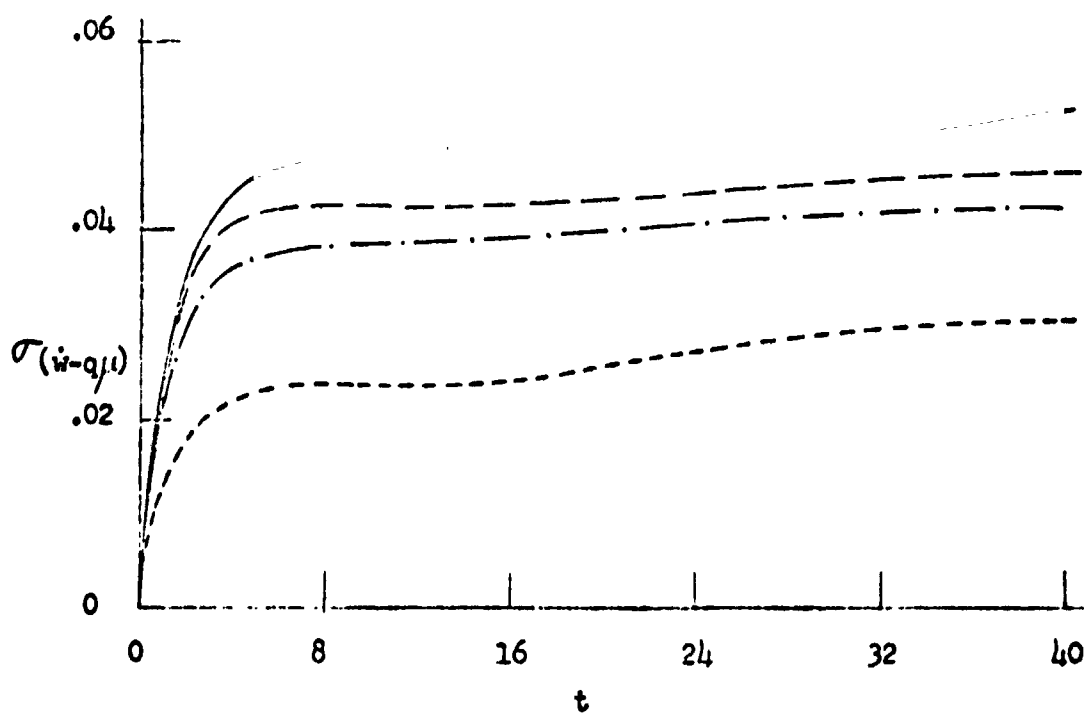
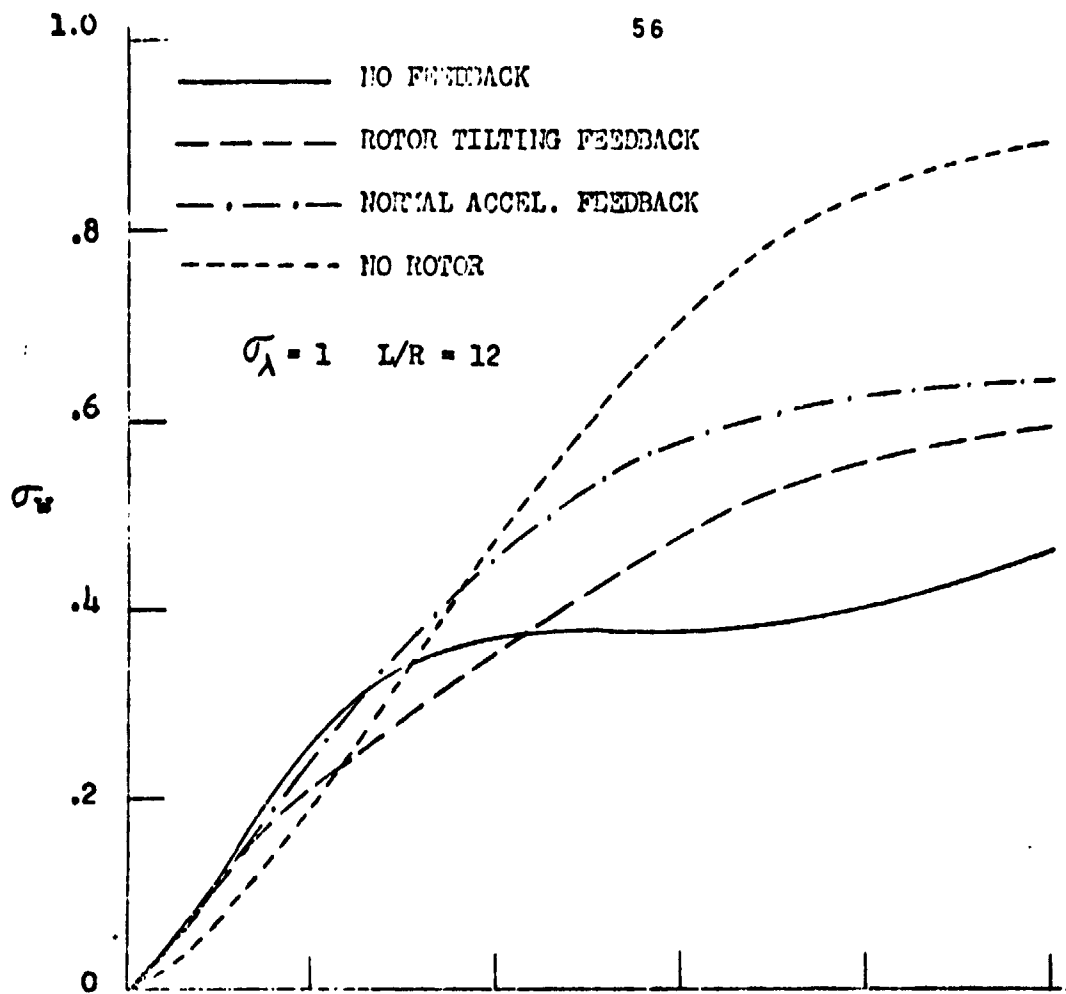


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